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Ministry of Education and
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# PHYSICS

## Second Secondary




2018 - 2019







# Physics

## Second Second Secondary


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# Foreword

Physics is the cornerstone of basic sciences. It deals with the understanding of nature and what goes around us, big and small in this universe. It is the root of all sciences. Interwined with it is chemistry which focuses on reactions between materials, biology which deals with living creatures, geology which is involved with the layers of the Earth, and astronomy which treats celestial objects. But in the end, physics remains the mother of all sciences and the basis for the tremendous present scientific and technological progress. Understanding physics means understanding the laws governing this universe. Such understanding has led to the current industrial development spearheaded by the West. The Arabs and Moslems were once the pioneers of civilization in the world when they realized the importance of understanding the laws of this universe. We owe them the discovery of most laws of physics centuries before the West. The foundations of medicine, physics, chemistry, astronomy, mathematics and music were all laid by Arab and Moslem scientists.

In fact, understanding physics and its applications converts a poor, and underdeveloped society into an affluent and developed one. This has taken place in Europe, US, Japan and South East Asia. Computers, satellites, cellular (mobile) phones, and TV are all byproducts of

physics. Genetics is currently being looked into intensively. It is targeted to use genetics, atoms and lasers in the computer of the future. It is a limitless world, enriched by imagination where sky is the limit.

The scientific progress is a cumulative effort. This collective endeevor has led to where we are today. A scholar of physics must be acquainted with such accumulated knowledge in a short time, so that he could add to it within the limited span of his life. In studying what others have found, we must skip details trials, and extract the end results and build on them. A global view is therfore, more important at this stage than being drowned in minute details that could be postponed to a later stage of study.

This book is divided into 3 units. Unit 1 deals with waves, which are the basis of communication in the universe. (Chapter 1) deals with wave motion, and (chapter 2) with light.
Unit 2 deals with fluid mechanics,: hydrostatics (chapter 3) and hydrodynamics (chapter 4). Unit 3 deals with heat, where (chapter 5) deals with gas laws

**The book is provided with some links in Egyptian Knowledge Bank**
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**to enhanc learning and achieve better understanding**

# Unit 1

# Unit 1

# Waves

Chapter 1 : Wave Motion

Chapter 2 : Light

# Waves

Unit 1



## Chapter 1 : Wave Motion

# Chapter 1 Wave Motion

## Overview :

Many of us enjoy watching waves on the surface of water pushing a fishing float or a boat up and down, or even making waves by throwing a pebble in a pond or still water. Each pebble becomes a source of disturbance in the water, spreading waves as concentric circles (Fig 1-1). Hence, waves are disturbances that spread and carry along energy.



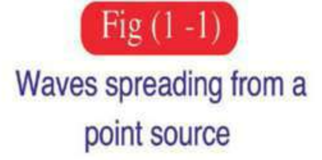


Fig (1 -1)
Waves spreading from a point source

Waves are not only water waves. There are, for example, radio waves. We often hear the announcer say: "This is Radio Cairo on the medium wave 366.7 m". Also, TV stations transmit both sound and image in the form of waves which are received by the aerial (antenna) . Such waves are transformed into electrical signals in the receiver, where they are eventually converted back to sound (audio) and image (video). Also, the mobile phone runs on waves. Sound signals are transformed into electrical signals then into

electromagnetic waves spreading in space and the surrounding medium . When received by the mobile antenna at the receiver, electromagnetic waves are transformed back into electrical signals and then to sound or even to an image.

We can see water waves but we cannot see the radio, TV or mobile waves. However, we can detect them. Water waves are mechanical waves, so are sound waves and waves in vibrating strings. But radio, TV, and mobile waves are electromagnetic waves. Among these electromagnetic (em) waves, there are, for example, light waves and X-rays which are used in radiology. Mechanical waves require a medium to propagate through, while (em) waves do not require a medium. They can propagate in space.



**Go Further**

For more knowledge about this topic you can refer to the Egyptian Knowledge Bank (EKB) through the opposite link



## Mechanical Waves

Mechanical waves require the following :-

1 ) a vibrating source.

2 ) a disturbance transmitted from the source to the medium.

3 ) a medium that carries a vibration.

There are many forms of vibrating sources :

1 ) a simple vibrating pendulum (Fig 1 - 2).

2 ) a tuning fork (Fig 1 - 3).

3 ) a vibrating stretched wire (or string) (Fig 1- 4).

4 ) a plumb (bob) attached to a vibrating spring (Yoyo) (Fig 1 - 5).



Fig (1 -2)
A pendulum

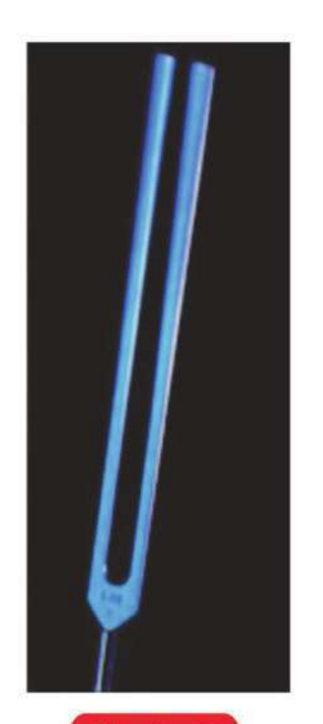


Fig (1 -3)
A tuning fork



Fig (1 -4)
A vibrating string

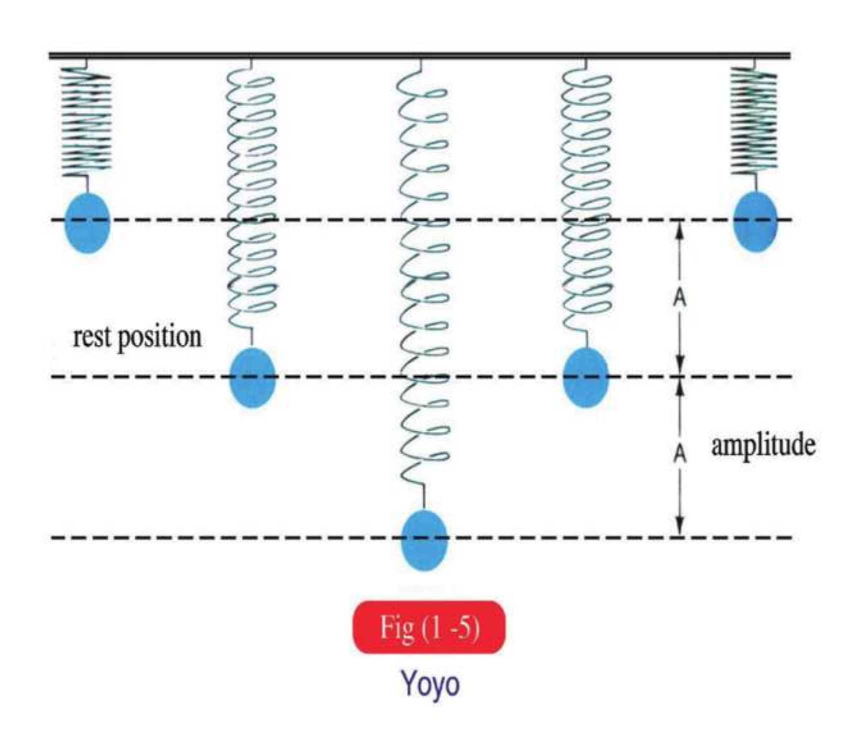



Fig (1 -5)

Yoyo

To study vibrations, we need to define some relevant physical quantities such as: displacement, amplitude, complete oscillation, periodic time and frequency as follows:

**1 - Displacement (meter):** is the distance of a vibrating body at any instant from its rest position or its equilibrium origin. It is a vector quantity.

**2-Amplitude (A) (meter):** is the maximum displacement of the vibrating object or the distance between two points along the path of the object, where the velocity at one point is maximum and zero at the other.

**3-Complete Oscillation:** is the motion of a vibrating body in the interval between the instants of passing by one point along the path of its motion twice successively with motion in the same direction and same displacement, i.e., at the same phase, relative to the starting point of motion.

**4-Frequency ($\nu$) (Hertz or Hz):** is the number of complete oscillations made by a vibrating body in one second.

**5-Periodic Time (T) (seconds):** is the time taken by a vibrating body to make one complete oscillation, or the time taken by the vibrating body to pass by the same point along the path of motion twice successively with motion in the same direction and the same displacement.

$$\nu = \frac{1}{T} \quad (1-1)$$

## Longitudinal Waves:

Imagine a mass "m" on a smooth horizontal surface attached to one end of a spring whose other end is attached to a vertical wall. If we pull the mass in the direction of the spring and let it go, the mass moves around its rest position in an oscillatory motion toward the spring and away (Fig 1-6). This is a simple harmonic motion. If we draw the curve that the center of gravity of the mass makes with respect to its rest position, we will obtain a sine wave (Fig 1-7). This is what distinguishes a simple harmonic motion from any other type of motion.

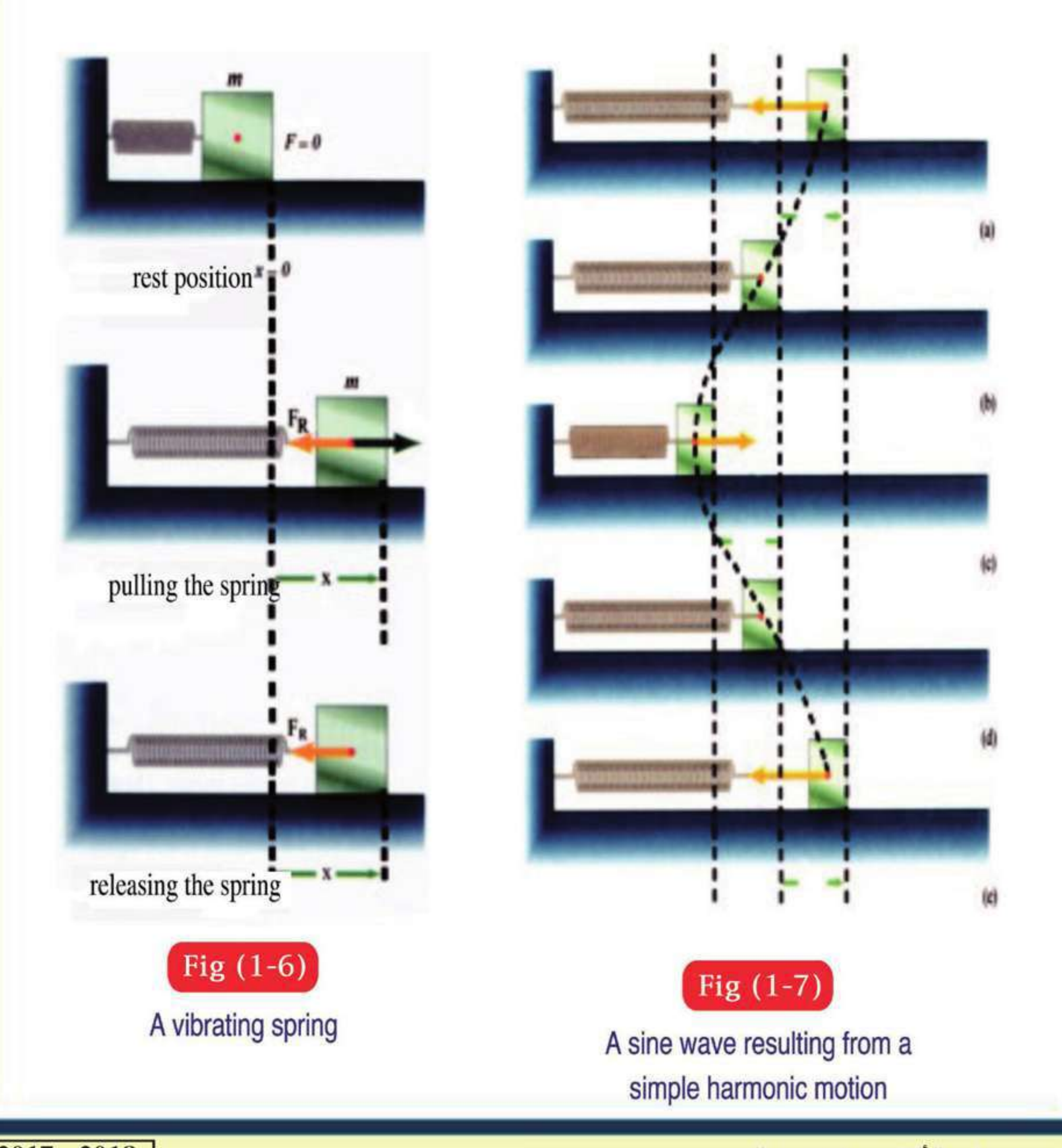



Fig (1-6)
A vibrating spring

Fig (1-7)
A sine wave resulting from a simple harmonic motion

Let us now imagine a mass "m" on a smooth horizontal surface attached at one end to a spring and the other end to another long spring, whose far end is attached to a vertical wall (Fig 1-8a). If we pull the mass "m" to the right in the direction of the spring to position x = A, then part of the spring to the right of "A" is compressed. This compression is transmitted successively to the right. If the mass "m" is pulled to the position x = - A, the spring to the right of the mass elongates leading to rarefaction. This rarefaction soon spreads to the right, when the mass m goes back to the rest position x = 0 again. The successive compressions and rarefactions form a wave motion performed by the vibrating particles of the medium (spring), which gives a simple harmonic motion. The direction of wave propagation is the same as the direction of the spreading of disturbance. This is called a longitudinal wave, since compressions and rarefactions spread along the length of the spring (Fig 1-8b).

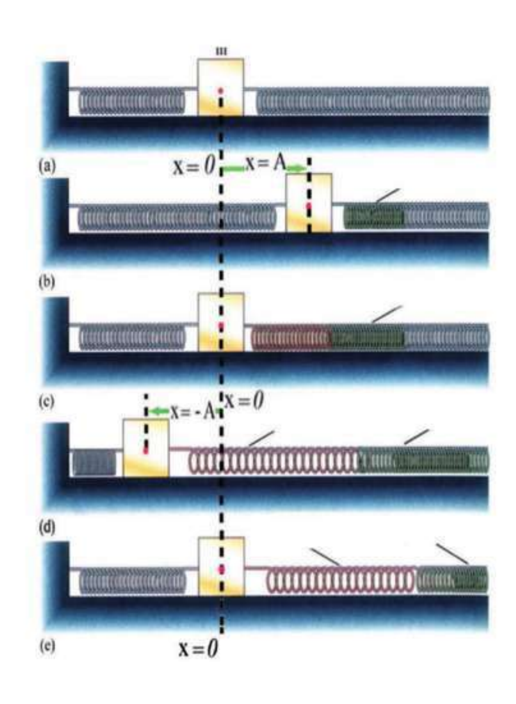



Fig (1-8a)

Compressions and rarefactions in a longitudinal wave

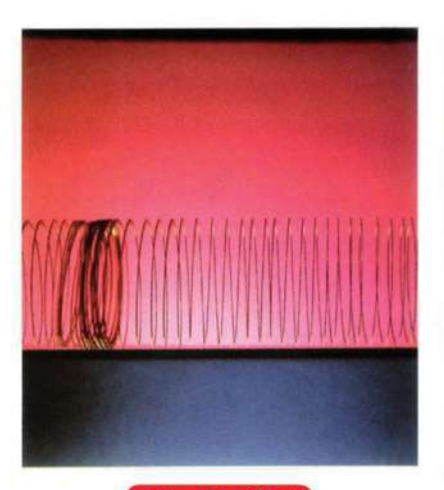


Fig (1-8b)

A vibrating spring forms a longi tudinal wave

Thus, a vibrating source making a simple harmonic motion may generate a wave propagating at velocity "v". Each particle of the medium performs, in turn, a simple harmonic motion about its equilibrium position. An example of this motion is the longitudinal waves of sound in air.

## Transverse Waves:

Imagine a mass "m" attached to a vertical spring. A long horizontal taut (stretched) rope is also attached to this mass at the near end, while the other (far) end of the rope is attached to a vertical wall.

When the mass "m" performs a simple harmonic motion in the vertical direction, then the near end of the rope performs the same motion. Consequently, the following parts of the rope do the same thing successively. Then the motion transfers horizontally along the rope in the form of a wave at velocity "v", while the other parts of the rope oscillate vertically in a simple harmonic motion about their rest positions. This wave is called a transverse wave (Fig 1-9a).

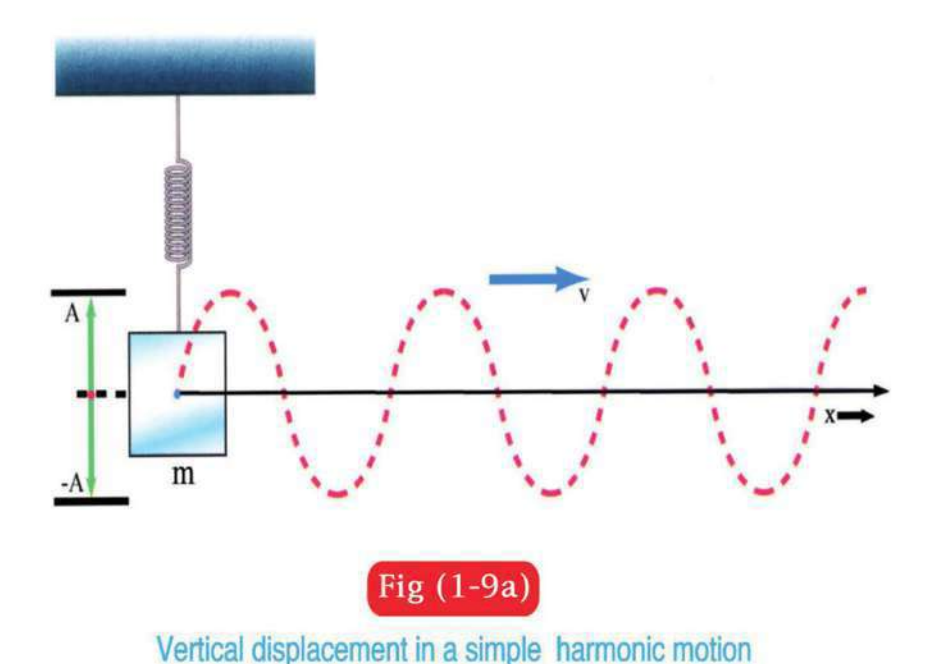



Fig (1-9a)

Vertical displacement in a simple harmonic motion

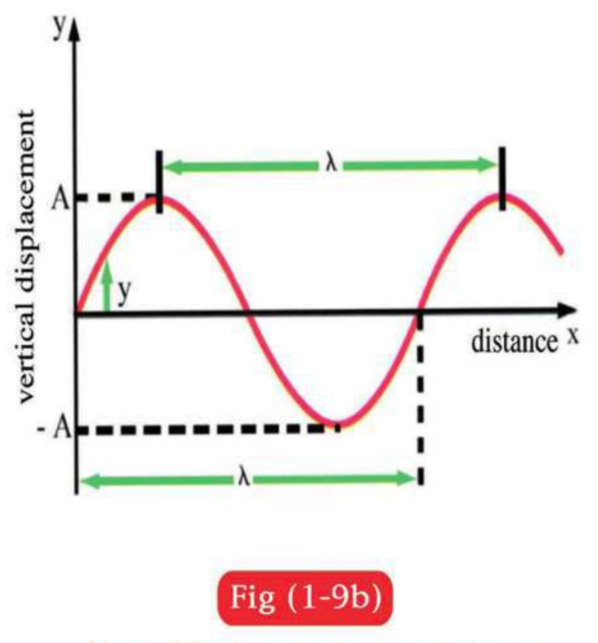



Fig (1-9b)

Vertical displacement as a sine wave

You can do this experiment yourself by using a long stretched rope. The far end is attached to a vertical wall while the near end is in your hand. When you move your hand up and down in the form of a pulse, you note that the wave spreads in a pulse form along the rope. This is known as a traveling wave (Fig 1-10).

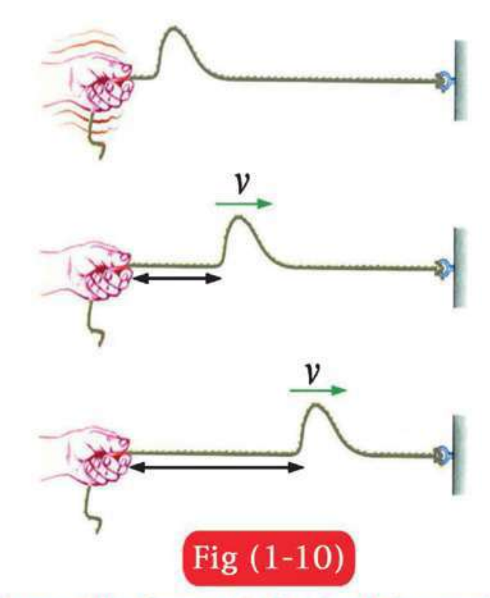



Fig (1-10)

A pulse resulting from part of a simple harmonic motion spreading along a stretched rope

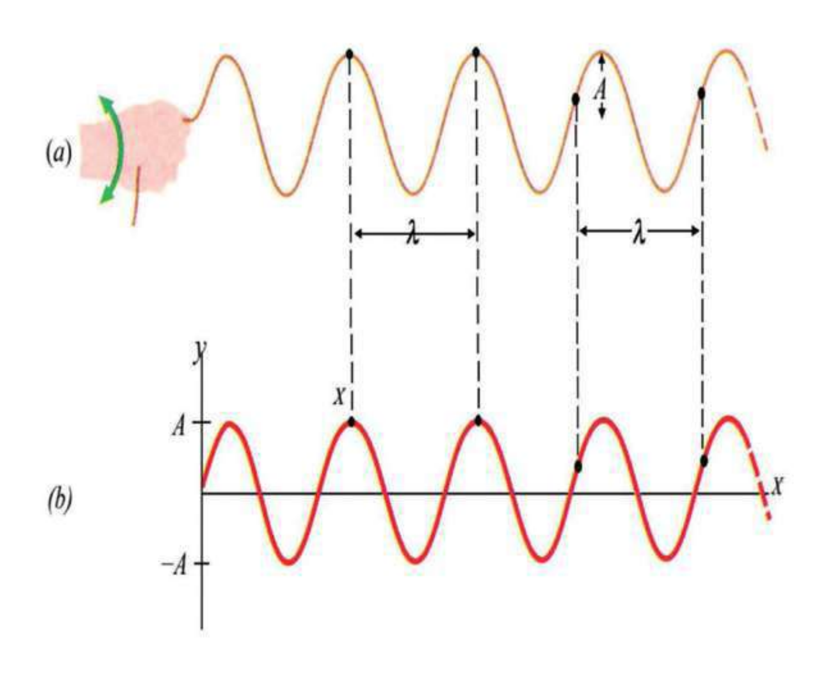



Fig (1-11)

A train wave spreading in a taut (stretched) rope due to a continuous simple harmonic motion at the near end

A wave may also be continuous (called a traveling wave train) as long as the simple harmonic motion of the source keeps on (Fig 1-11).

The stretched rope may be replaced by a spring in which a longitudinal wave (Fig 1-12) or a transverse wave (Fig 1-13) may be generated . We conclude that as a source oscillates , the particles of the medium oscillate successively in the same way. The vibration transfers first from the source to the particle of the medium next to it, then into the one connected to it, then into the following ones and so on. Thus, the vibration or disturbance forms a wave, since the wave is nothing but a disturbance (or energy) on the move along which energy is carried through .

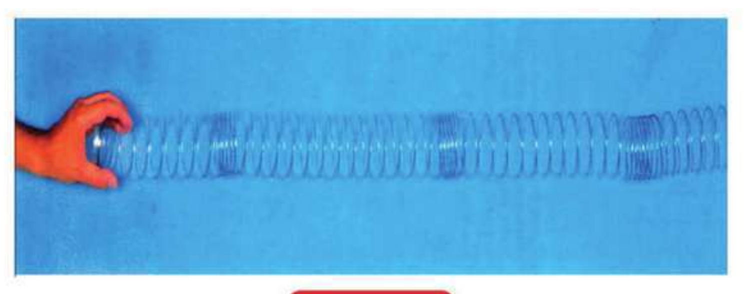


Fig (1-12)
A vibrating spring forming a longitudinal wave

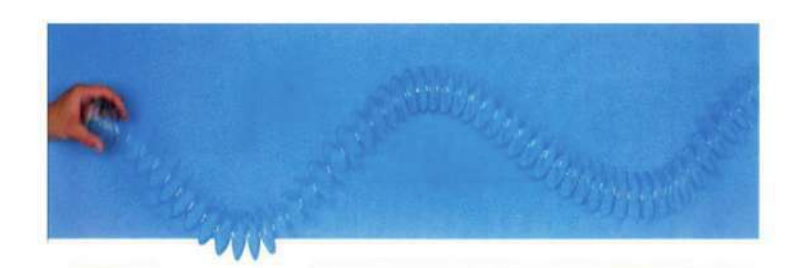


Fig (1-13)
A vibrating spring forming a transverse wave

In conclusion, we may classify mechanical waves into two types:

1 ) Transverse waves

2 ) Longitudinal waves

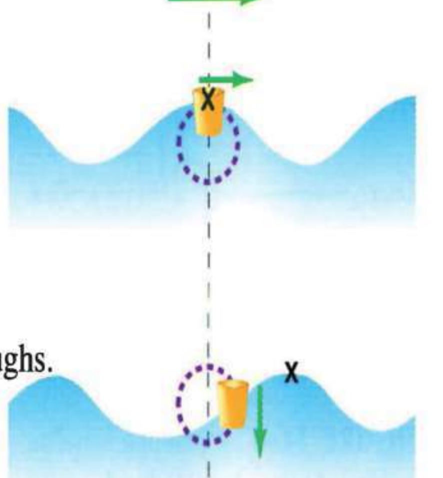
In transverse waves, the particles of the medium oscillate about their equilibrium positions in a direction perpendicular to the direction of the propagation of the wave.

In longitudinal waves, the particles of the medium oscillate about their equilibrium positions along the direction of the propagation of the wave.

The work done by the oscillating source is converted to the particles of a string (or a stretched rope) in the form of potential energy stored as tension in the string and kinetic

energy manifested in the vibration of the particles of the string.

direction of wave propagation



Referring to (Fig 1-14), the points at maximum upward displacement in the positive direction are called crests, while the points of maximum downward displacement **in the negatived direction** are called troughs.

Observing any part of a vibrating string carrying a transverse wave, we find that it has one crest and one trough during one complete oscillation .



Fig (1-14)

A piece of foam floating on the top of a wave (crest) or at bottom (trough)

## Frequency ($\nu$) (Hertz) and wavelength ($\lambda$) (meter):

The distance between two successive crests or two successive troughs in a transverse wave is called wavelength (Fig 1-15). Similarly, the distance between two successive contractions (compressions) or two successive rarefactions in a longitudinal wave is called wavelength (Fig 1-16).

Thus, we may represent the wavelength by either of the two distances (AC) and (BD)(Fig 1-17). It is to be noted that the two successive pairs of points (A,C) and (B, D) move in the same way at the same time. We say they have the same phase, i.e.,the same displacement in the same direction.

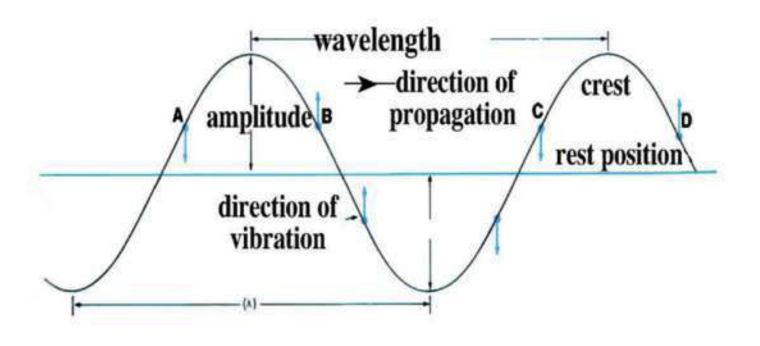



Fig (1-15)

Wavelength in a transverse wave

Thus, the wavelength is the distance between two successive points of the same phase (Fig 1-17). Alternatively, it is the distance which the wave travels during one periodic time (Fig 1-18).

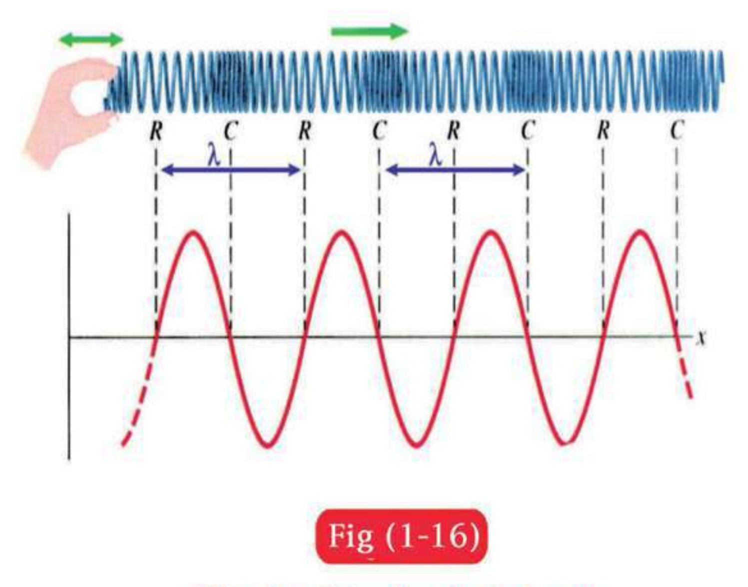



Fig (1-16)

Wavelength in a longitudinal wave

The number of waves passing by a certain point along the wave path in one second is called frequency.

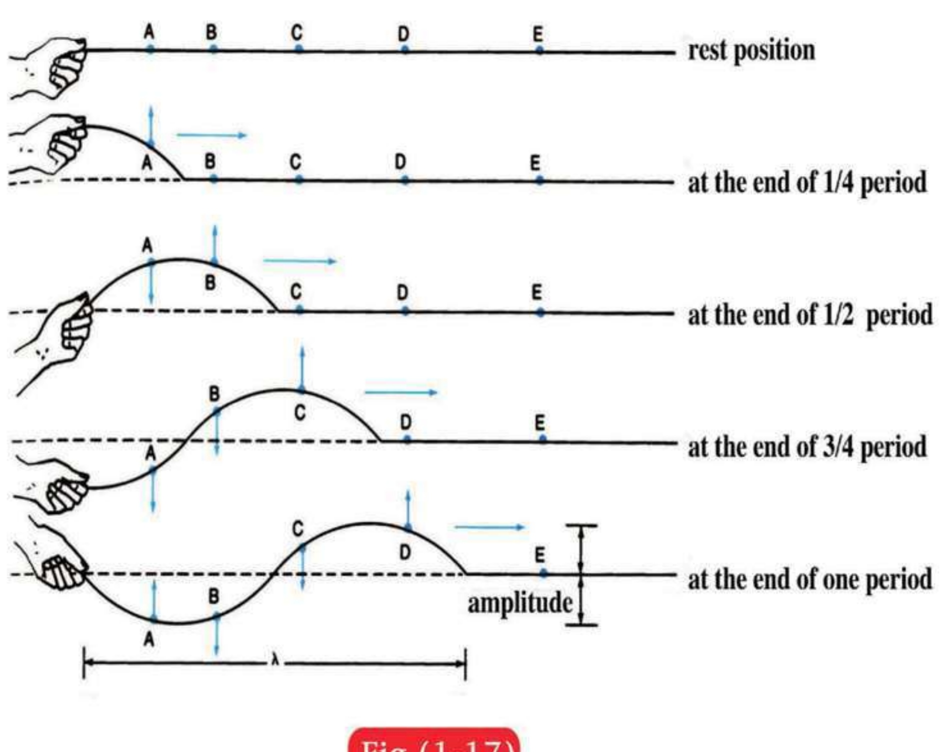



Fig (1-17)

The distance after each one full vibration completed in one period T is the wavelength

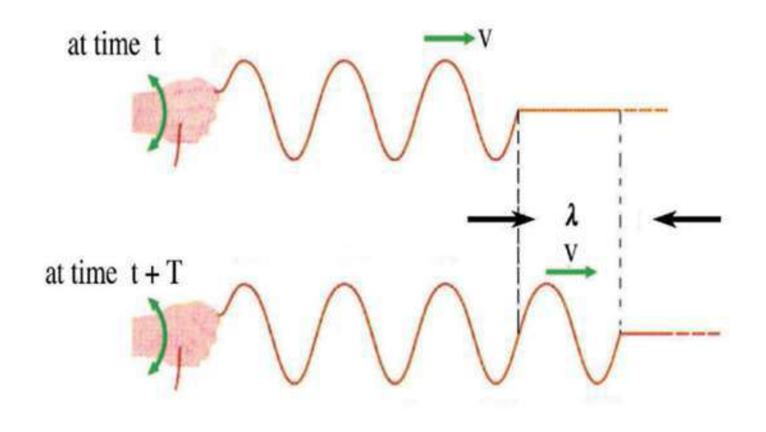



Fig (1-18a)

The distance which a wave moves in a periodic time T is the wavelength

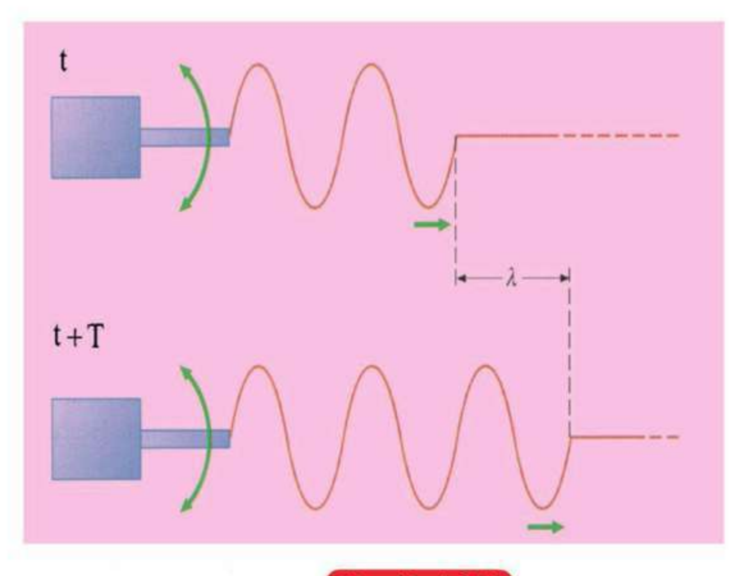



Fig (1-18b)

A train of waves at velcity "v" generated by a vibrator

## The relation between frequency,wavelength and velocity of propagation:

If a wave travels at velocity "v", a distance equal to the wavelength "λ" , then the wave takes time equal to the periodic time "T" to travel this distance.

$$v = \frac{\lambda}{T}$$

$$\nu = \frac{1}{T}$$

$$T = \frac{1}{\nu}$$

$$v = \lambda \nu$$

This is a general relation for all types of waves.

In all cases , within a periodic time "T" a wave travels a wavelength.

Frequency is the number of oscillations in one second or the number of wavelengths traveled by a wave propagating in a certain direction in one second.

## Examples:-

1) If the wavelength of a sound wave produced by a train is 0.6 m and the frequency is 550 Hz,what is the velocity of sound in air?

**Solution:-**

$$v = \lambda \nu$$

$$v = 0.6 \times 550 = 330 \quad m/s$$

2) If the number of waves passing by a certain point in one second is 12 oscillations and the wavelength is 0.1 m, calculate the speed of propagation.

**Solution:-**

$$v = \lambda \nu$$

$$v = 12 \times 0.1 = 1.2 \quad m/s$$

3) Light waves propagate in space at speed 300 000km/s ($3\times10^8$m/s), and the wavelength of light is 5000 A° .What is the frequency of this light ?

1 Angstrom($A^0$) $=10^{-10}$ m

**Solution:-**

$$c = v = 3 \times 10^8 \quad m/s$$

$$\lambda = 5 \times 10^3 \times 10^{-10} = 5 \times 10^{-7} \ m$$

$$c = \lambda \nu$$

$$3 \times 10^8 = 5 \times 10^{-7} \times \nu$$

$$\nu = \frac{3 \times 10^8}{5 \times 10^{-7}} = 6 \times 10^{14} \ Hz$$

### Go Further

For more knowledge about this topic you can refer to the Egyptian Knowledge Bank (EKB) through the opposite links




## In a Nutshell

- A wave is a disturbance which spreads and carries energy along.
- Displacement is the distance of an object at any instant from its rest(equilibrium) position.
- The amplitude of oscillation "A" is the maximum displacement of an oscillating object from its rest position, or the distance between two points along the path of the oscillating object where the velocity at one point is maximum and at the other is nil.
- A complete oscillation is the movement of a continuously vibrating body ( e.g. a simple pendulum) is the interval between the instants of time as it passes by a certain point along its path twice successively with motion in the same direction.
- Frequency "ν" is the number of complete oscillations produced by a vibrating object in one second and is equal to the inverse of the periodic time.

$$\text{Frequency} = \frac{1}{\text{Periodic time}}$$

It is also the number of waves passing by a certain point along the path of a wave in one second.

- Periodic time "T" is the time taken by a continuously vibrating body to perform one complete oscillation, or the time taken by a continuously vibrating body ( e.g. a simple pendulum ) to pass by a point along its path twice successively with motion in the same direction.
- Mechanical waves are either:
  1 ) transverse waves.
  2 ) longitudinal waves.
- Transverse waves are waves in which the particles of a medium oscillate about their equilibrium positions in a direction perpendicular to the direction of propagation of the wave.
- Longitudinal waves are waves in which the particles of a medium oscillate about their equilibrium positions along the same path of propagation of the wave.
- Transverse waves comprise crests and troughs in succession.

- Longitudinal waves comprise compressions and rarefactions in succession .
- Wavelength is the distance between two successive points along the direction of propagation of the wave, where the phase is the same (same displacement and same direction).
- The relation between frequency, wavelength and velocity of a wave is given by: $v = \lambda \nu$

## Questions and Drills

**I) Define**

Wave - Transverse Wave - Longitudinal Wave - Wavelength

**II) Complete:**

a) Displacement is ......

b) Amplitude of oscillation is ......

c) Complete oscillation is ......

d) Periodic time is ......

e) Frequency is ......

**III) Essay question:**

Deduce the relation between frequency, wavelength and velocity of wave propagation.

**IV) Put a tick sign (√) next to the right choice in the following :**

1) The relation between the velocity of propagation of the waves "v" in a medium , its frequency and wavelength is :

a) $v = \lambda \nu$ b) $v = \nu / \lambda$

c) $v = \dfrac{\lambda}{\nu}$ d) there is no correct answer

2) Transverse waves are waves consisting of :

a) Compressions and rarefactions

b) Crests and troughs

c) Crests and troughs, where the particles of the medium move short distances about their equilibrium positions in a direction perpendicular to the direction of propagation.

d) Compressions and rarefactions, where the particles of the medium move short distances about their equilibrium positions along the direction of propagation of the wave .

3) If the wavelength of a sound wave produced by an audio ( sound producing) source is 0 . 5 m , the frequency is 666 Hz ,then the velocity of propagation of sound in air is :

a) 338 m / s b) 333 m / s c) 330 m / s d) 346 m / s

4) If the velocity of sound in air is 340 m/s, for a sound of frequency (tone) 255 Hz , the wavelength(m) is :

a) 4/3 b) 3/4 c) 2 d) 3/2

5) Light of wavelength 6000 Å (1Å = $10^{-10}$m ) propagates in space at velocity 300 x $10^{3}$km/s, its frequency is:

(a) 4 x $10^{10}$ Hz (b) 4 x $10^{14}$ Hz

(c) 5 x $10^{14}$ Hz (d) 5 x $10^{12}$ Hz

6) Two waves whose frequencies are 256 Hz and 512 Hz propagate in a certain medium , the ratio between their wavelengths is

a) 2/1 b) 1/2 c) 3/1 d) 1/3

# Chapter 2 Light

## Overview :

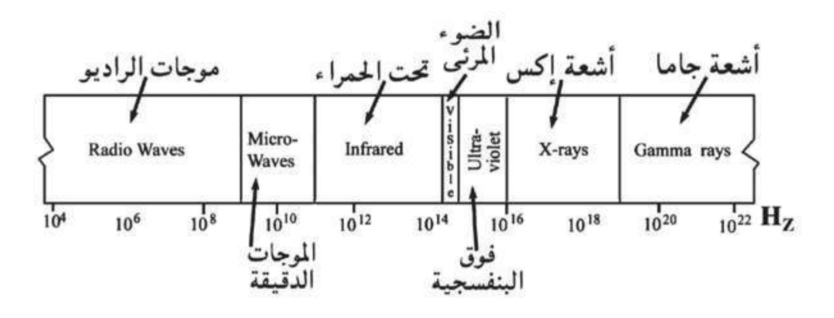



Fig (2-1)

Electromagnetic Spectre

Light is part of an extensive range of waves called electromagnetic waves, which all travel at a constant speed in space equal to $3 \times 10^8$ m/s, while varying in frequency. This range of waves is called the electromagnetic spectrum (Fig 2-1). It includes, for example, radio waves, infrared, visible light, ultraviolet, X- rays and Gamma rays. They all share common features indicated in the link below.



## Reflection and refraction of light

Light propagates in straight lines in all directions, unless met by an obstructing medium. If so, it undergoes reflection, refraction and partial absorption depending on the nature of the medium. When a light ray falls on a surface separating two media - which are different in optical density - then part of light is reflected and the rest is refracted, neglecting absorption. We note from Fig (2-2) that each of the incident ray, reflected ray and refracted ray as well as the normal to the surface at the point of incidence all lie in one plane perpendicular to the separating surface.

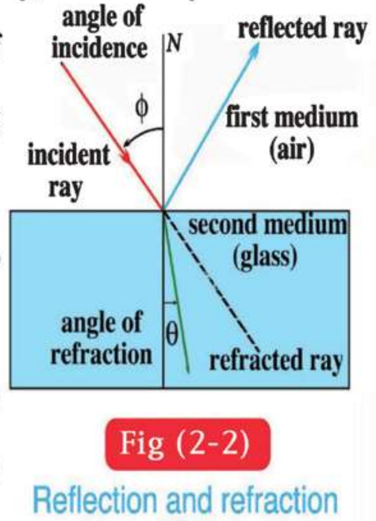



Fig (2-2)

Reflection and refraction of light

In the case of reflection : the angle of incidence is equal to the angle of reflection

In the case of refraction : the ratio between the sine of the angle of incidence in the first medium to the sine of the angle of refraction in the second medium is equal to the ratio

of the speed of light in the first medium to the speed of light in the second medium. This ratio is constant for these two media, and is called relative refractive index from the first medium to the second medium, denoted by ${}_1n_2$ :

$$\frac{\sin \phi}{\sin \theta} = \frac{v_1}{v_2} = {}_1n_2 \qquad (2-1)$$

## Important facts

1) The speed of light in space "c" is one of the physical constants of the universe and is equal to $3 \times 10^8$ m/s. It is larger than the speed of light in any medium "v". The ratio $\frac{c}{v} = n$ is called the absolute refractive index for the medium and is always > 1.

$$n = \frac{c}{v} \qquad (2-2)$$

The absolute refractive indices of some materials are listed below

| refractive index | medium |
|---|---|
| Air | 1.00293 |
| Water | 1.333000 |
| Benzine | 1.501000 |
| Carbon tetrachloride | 1.461000 |
| Ethyl alcohol | 1.361000 |
| Crown glass | 1.52000 |
| Rock glass | 1.660000 |
| Quartz | 1.4850000 |
| Diamond | 2.419000 |

2) **From the relation (2-2)**

$$v = \frac{c}{n}$$

$$\frac{v_1}{v_2} = \frac{n_2}{n_1} \qquad (2-3)$$

where $v_1$ is the speed of light in the first medium, $v_2$ is the speed of light in the second medium. Substituting equation (2-3) in (2-1), we have:

$$\frac{n_2}{n_1} = \frac{\sin\phi}{\sin\theta}$$

$$n_1 \sin\phi = n_2 \sin\theta \qquad (2-4)$$

This is Snell's law

The absolute refractive index for the medium of incidence times the sine of the angle of incidence is equal to the absolute refractive index of the medium of refraction times the sine of the angle of refraction.

3) We can use refraction in analyzing a bundle of light into its components of different wavelengths, since the absolute refractive index varies with wavelength. Therefore, white light may be decomposed into its components. This can be seen, for example, in soap bubbles.

## Go Further

For more knowledge about this topic you can refer to the Egyptian Knowledge Bank (EKB) through the opposite links





## Examples

1) If a light ray falls on the surface of a glass slab whose refractive index is 1.5 at an angle $30^\circ$, calculate the angle of refraction.

**Solution**

$$\because \quad n = \frac{\sin\phi}{\sin\theta}$$

$$\therefore \quad 1.5 = \frac{\sin 30}{\sin\theta}$$

$$\sin\theta = \frac{0.5}{1.5} = 0.333$$

$$\therefore \quad \theta = 19^\circ\, 28'$$

2) If the absolute refractive index of water is $\frac{4}{3}$ and glass $\frac{3}{2}$, find

a) the relative refractive index from water to glass

b) the relative refractive index from glass to water.

**Solution**

a) The relative refractive index from water to glass

$$ {}_1n_2 = \frac{n_2}{n_1} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8} $$

b) The relative refractive index from glass to water

$$ {}_2n_1 = \frac{n_1}{n_2} = \frac{\frac{4}{3}}{\frac{3}{2}} = \frac{8}{9} $$

From this example, we note:

## Interference of light

Thomas Young performed a well known experiment to study the interference of light, namely the double slit experiment (Fig 2-3). In this experiment, he used a monochromatic (single wavelength) light source, i.e., $\lambda$ has one value.

At an appropriate distance from the source, lies a screen with a rectangular slit S through which cylindrical waves pass toward another screen with two narrow slits $S_1$ and $S_2$, which act as a double slit, and lie on the wavefront of the cylindrical wave. Therefore, waves



Fig (2-3a)

Interference in Young's experiment

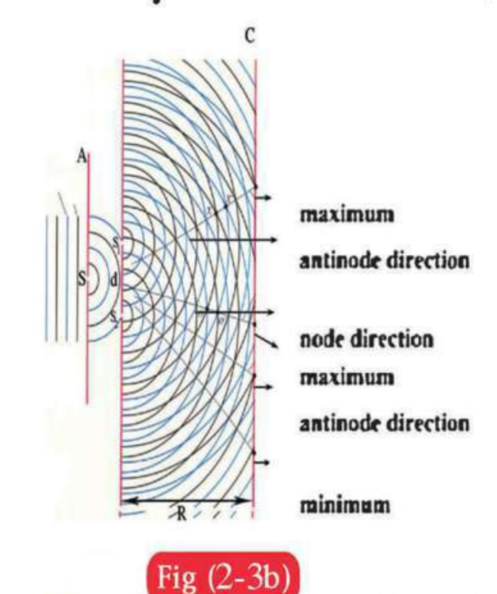



Fig (2-3b)

A schematic diagram for interference

reaching the double slit have the same phase, hence, are coherent (having the same frequency, amplitude and phase). Waves emanating from $S_1$ and $S_2$ are cylindrical and spread toward the observation screen C. On such a screen, waves coming from $S_1$ and $S_2$ combine and produce an interference pattern, appearing as a sequence of bright and dark straight parallel regions, which are the interference fringes (Fig 2-4). The distance between two successive fringes $\Delta y$ is given by:

$$\Delta y = \frac{\lambda R}{d} \qquad (2-5)$$

where $\lambda$ is the wavelength of the monochromatic source, R is the distance between the double slit and the observation screen and d is the distance between $S_1$ and $S_2$.

Therefore, this experiment may be used to determine the wavelength for any monochromatic light source.



Fig (2-4)
Interference fringes



## Go Further

For more knowledge about this topic you can refer to the Egyptian Knowledge Bank (EKB) through the opposite link



## Examples

In a double slit experiment, the distance between the slits is 0.00015m, the distance between the double slit and the observation screen is 0.75 m and the distance between two bright fringes is 0.003 m. Calculate the wavelength of the monochromatic light source.

**Solution**

$$\Delta y = \frac{\lambda R}{d}$$

$$0.003 = \frac{0.75 \times \lambda}{0.00015}$$

$$\lambda = \frac{0.00015 \times 0.003}{0.75} = 0.6 \times 10^{-6}\ m$$

$$10^{10} \times 0.6 \times 10^{-6} = 6000\ \text{Å}$$

## Light Diffraction

When a monochromatic light falls on a circular aperture in a screen, we expect that light should form a circular bright spot on an observation screen, considering that light propagates in straight lines. But careful examination of the bright spot (called Airy's disk), i.e., studying the light intensity, reveals the existence of bright and dark fringes (Fig 2-5).

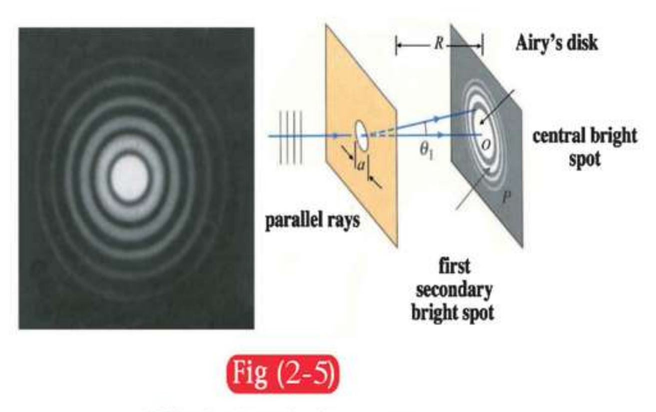



Fig (2-5)

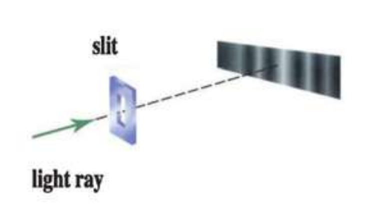
Diffraction in a circular aperture

(Fig 2-6) demonstrates diffraction from a rectangular slit, while In general, diffraction is evident when the wavelength of the wave is comparable to the dimensions of the aperture, and vice versa In fact, there is no big difference between the mechanisms of interference and diffraction. In both cases, combination (superposition) of waves is involved .

### Go Further

For more knowledge about this topic you can refer to the Egyptian Knowledge Bank (EKB) through the opposite link





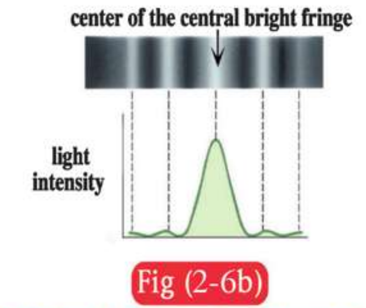




Fig (2-6b)

Distribution of light intensity on a screen with the succession of fringes resulting from diffraction from a rectangular aperture

Fig (2-6a)

Diffraction from a rectangular aperture

## Light as a wave motion

From above, we conclude that light

1) propagates in straight lines .
2) reflects according to the laws of reflection .
3) refracts according to the laws of refraction .
4) light interferes, and as a result, light intensity increases in certain positions (bright fringes)and diminishes to zero in other positions (dark fringes).
5) light diffracts if obstructed by an obstacle .

These are the same general properties of waves. Hence, light is a wave motion

## Total reflection and the critical angle

When a light ray travels from an optically dense medium (as water or glass) to a less dense medium (as air), then the refracted ray deviates away from the normal (Fig 2-7). As the angle of incidence increases in the more dense medium (of high absolute refractive index), the refraction angle in the less dense medium (of low absolute refractive index) increases.

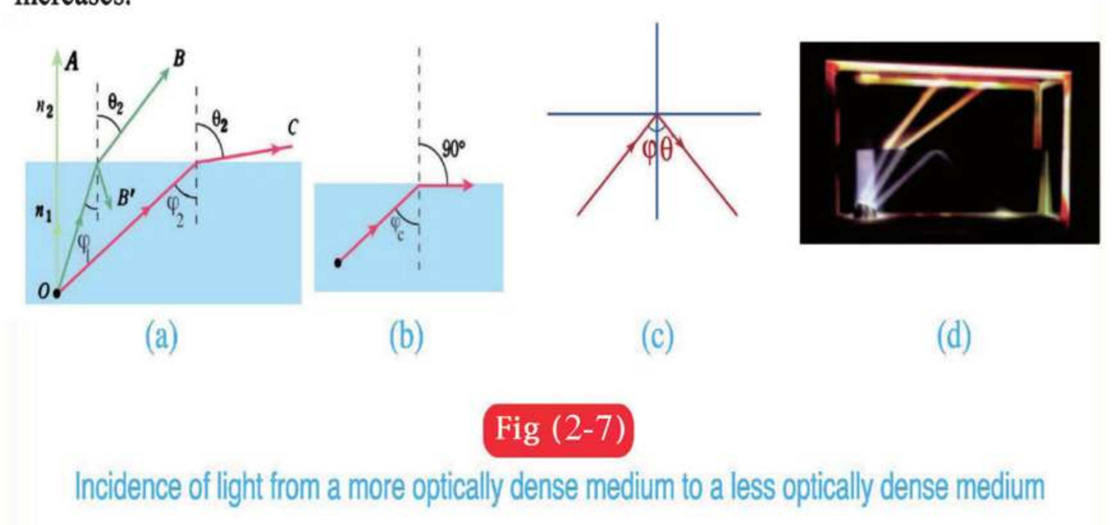



Fig (2-7)

Incidence of light from a more optically dense medium to a less optically dense medium

A point is reached when the angle of incidence in the more dense medium approaches a critical value $\phi_c$ when the angle of refraction in the less dense medium reaches its maximum, which is 90°. Then,the refracted ray becomes tangent to the surface.

We call this a grazing ray, and this angle of incidence is called the critical angle, which is the angle of incidence in the dense medium, which corresponds to an angle of refraction in the less dense medium equal to 90°.

$$n_1 \sin \phi_c = n_2 \sin 90°$$

$$\sin \phi_c = \frac{n_2}{n_1} = {}_1n_2 \qquad (2-6)$$

where $n_1$ is the refractive index of the more dense medium and $n_2$ is the refractive index of the less dense medium. In the case when the less dense medium is air $n_2 = 1$.

Thus, we can calculate the refractive index of a medium by knowing its critical angle.

$$n_1 \sin \phi_c = 1$$

$$n_1 = \frac{1}{\sin \phi_c} \qquad (2-7)$$

If the angle of incidence in the more dense medium exceeds the critical angle, then the light ray does not transmit through to the less dense medium, but undergoes total reflection in the same medium, unlike the case when the angle of incidence is less than the critical angle, where part of the rays is reflected and part is transmitted (Fig 2-7). (neglecting absorption)

**Examples:**

**1)** If the refractive index of glass and water are 1.6 and 1.33, respectively, calculate the critical angle for each

**Solution**

**a)** In the case of glass,

$$n_1 = \frac{1}{\sin \phi_c}$$

$$\sin \phi_c = \frac{1}{n_1} = \frac{1}{1.6} = 0.625$$

$$\phi_c = 38° 41'$$

b) In the case of water

$$\sin\phi_c = \frac{1}{n_1} = \frac{1}{1.33} = 0.7518$$

$$\phi_c = 48^\circ 45'$$

2)Using the information in the example above, find the critical angle for light falling from glass onto water

**Solution**

Using Snell's law,

$$n_2 \sin 90^\circ = n_1 \sin\phi_c$$

$$1.33 \times 1 = 1.6 \sin\phi_c$$

$$\sin\phi_c = \frac{1 \times 1.33}{1.6} = 0.8313$$

$$\therefore \phi_c = 56^\circ 14'$$

## Some Applications of Total Reflection

### I. Fiberoptics (Optical Fibers)

**(Fig 2-8).** shows an optical fiber. It is a thread-like tube of transparent material. When light falls at one end, while the angle of incidence is greater than the critical angle, it undergoes successive multiple reflections until it emerges from the other end **(Fig 2-9)**. **(Fig 2-10)** shows a bundle of fibers which can be bent while containing light so that light can be made to travel in non straight lines to parts hard to reach otherwise.

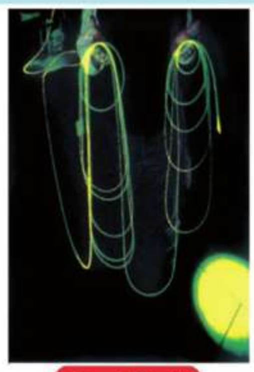


Fig (2-8)

Optical fibers contain the rays despite bending

Fibers can be used to transmit light without much losses, and are widely used nowadays.

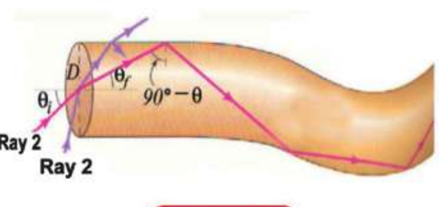



Fig (2-9)

Optical fibers

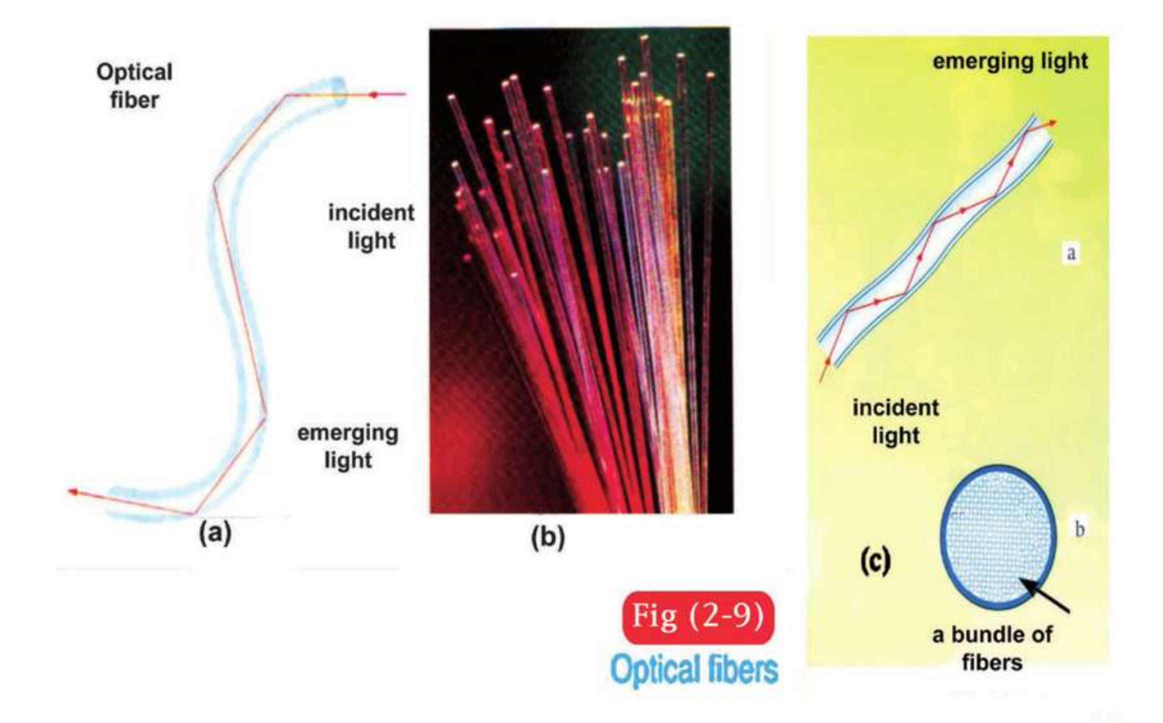



Fig (2-9)

Optical fibers

They are used in medical examinations, e.g., endoscopes (Fig 2-11). which are used in diagnosis as well as in operative surgery with a laser beam Lasers are also used in communications, as light can be made to carry millions of electrical signals in fiberoptic cables .





Fig ( 2-11)

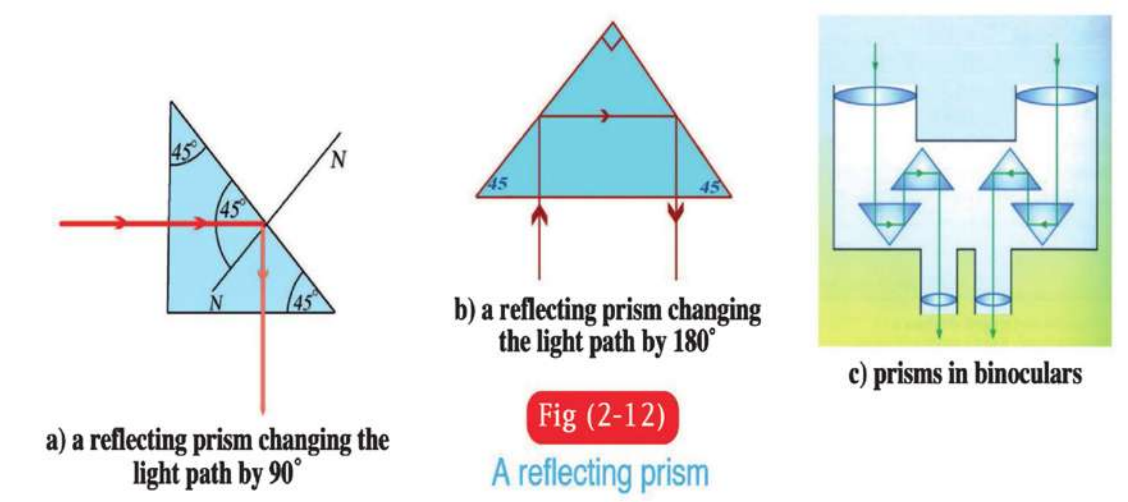
Endoscopes

### *How an optical fiber works*

If we have a hollow tube and look through it to see a bright object on the other end, then the object is easily seen. If the tube is bent, then the object cannot be seen. Yet, we may be able to see it, if we use reflecting mirrors in the path of rays. Similarly, by using optical fibers, while the ray is incident at an angle greater than the critical angle, then multiple reflections take place, until the ray emerges from the other end, despite the bending of the fibers.

## II. The reflecting prism

The critical angle between glass (refractive index 1.5) and air is 42°. Therefore, a glass prism whose angles are 90°, 45°, 45° is used to change the path of the rays by 90° or 180°. Such a prism may be used in optical instruments, as periscopes in submarines and binoculars in the field (Fig 2-12).



a) a reflecting prism changing the light path by 90˚

b) a reflecting prism changing the light path by 180˚

c) prisms in binoculars

Fig (2-12)
A reflecting prism

Prisms are better for this purpose than reflecting surfaces, first, because light totally reflects from such a prism, while it is seldom to find a metallic reflecting surface whose efficiency is 100%. Secondly, a metallic surface eventually loses its luster,and hence its ability to reflect decreases. This does not happen in a prism. The surface at which light rays fall on a prism or the surface from which the rays emerge may be coated with non - reflective layer of material like cryolite (Aluminum fluoride and magnesium fluoride) whose refractive index is less than that of glass, to avoid any reflection losses on the prism, even little as they are.

## III.Mirage

This is a familiar phenomenon observable on hot days, as paved roads appear as if wet (Fig 2-13a). Also, an image of the sky is made on desert plains, where palm trees or hills appear inverted giving the illusion of water (Fig 2-13b).



Fig (2-13a)
Paved roads appear as if wet

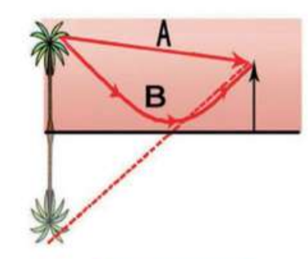



Fig (2-13b)
Reflection of the sky in the desert gives the illusion of water

This can be explained as follows. On very hot days, the air layers adjacent to the surface of the Earth are heated, their density decreases. Hence, their refractive index becomes smaller than that of the upper layer. If we follow a light ray reflected off a palm tree, this ray is traveling from an upper layer to one below. Therefore,if refracts away from the normal, and keeps deviating taking a curved path. When its angle of incidence reaches more than the critical angle, it undergoes total reflection and the curve goes up. To the eye of the observer, the ray appears as if coming from under the surface of the Earth. The observer thinks that there is a pond.

## Deviation of light in a prism

When a light ray such as " a b " falls on the surface xy of a triangular prism, it refracts in the prism taking the path " bc ", until it falls on the surface xz and emerges in the direction "cd" (Fig 2-14). We notice from the figure that the light ray in the prism refracts twice. As a result, the ray deviates from its original path by an angle of deviation $\alpha$.

The angle of deviation $\alpha$ is the angle subtended by the directions of the extension of the incident ray and the emerging ray. If the angle of incidence is $\phi_1$, the angle of refraction on the first surface is $\theta_1$, the angle of incidence on the second surface is $\phi_2$, the angle of emergence is $\theta_2$ and the apex angle of the prism is A, we note from the geometry (Fig 2-14).

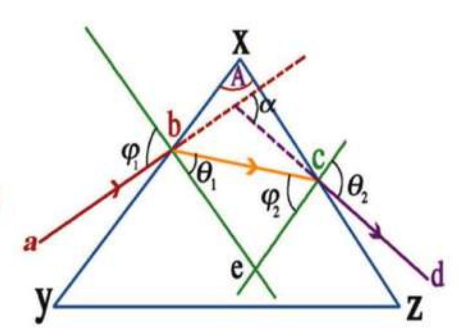



Fig (2-14)

The path of a light ray in a triangular prism

The sum of the angles of the quadrilateral bxce is 360°, and the sum of the two angles b,c is 180°, then angle $b\hat{e}c$ is (180°-A). In the triangle bce, the sum of the angles is 180°. Thus,

Thus,

$$\theta_1 + \phi_2 = 180° - (180° - A°) = A° \qquad (2-8)$$

$$\alpha = (\phi_1 - \theta_1) + (\theta_2 - \phi_2) = (\phi_1 + \theta_2) - (\theta_1 + \phi_2)$$

$$\alpha = \phi_1 + \theta_2 - A° \qquad (2-9)$$

From this relation, we find that the deviation angle in a triangular prism depends on the angle of incidence $\phi_1$. It can be experimentally shown that the deviation angle decreases gradually with increasing $\phi_1$ until it reaches a minimum known as the minimum angle of deviation $\alpha_\circ$ (Fig 2-15).

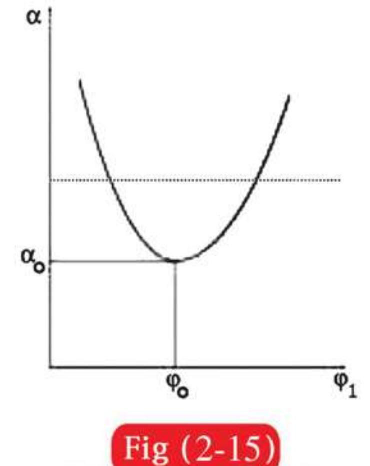



Fig (2-15)

The angle of deviation has a minimum

At this minimum

$$\phi_1 = \theta_2 = \phi_\circ$$

$$\theta_1 = \phi_2 = \theta_\circ$$

Then, the relations (2-8) and (2-9) become

$$A = 2\theta_\circ$$

or $$\therefore \quad \theta = \frac{A}{2}$$

and $$\alpha_\circ = 2\phi_\circ - A$$

or $$\therefore \quad \phi_\circ = \frac{\alpha_\circ + A}{2}$$

But $$n = \frac{\sin\phi_0}{\sin\theta_0}$$

Substituting for $\phi$ and $\theta$ we find that the refractive index can be determined from the relation

$$n = \frac{\sin\left(\frac{\alpha_0 + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad (2\text{-}10)$$

## Experiment to determine the ray path through a glass prism and its refractive index: (Fig 2-16).

### Tools:

An equilateral triangular prism (A = 60°), pins, a protractor, a ruler.

### Procedure:

1) Place the glass prism on a sheet of drawing paper with its surface in a vertical position and mark its position with a fine pencil line.

Place two pins such that one of them (a) is very close to one side and the other (b) is about 10 cm from the first. The line joining them represents the incident ray. Look at the other side of the prism to see the image of the two pins, one behind the other.

Place two other pins c and d between the prism and the eye such that they appear to be in line with the two pins a and b,i.e., the four pins appear to be in one straight line.Locate the positions of the four pins.

2) Remove the prism and the pins, join b and c to locate the path of the ray (a b c d) from air to glass to air again.

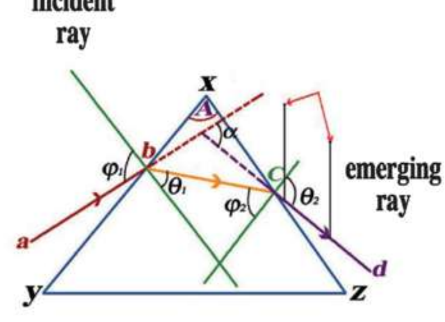



Fig (2-16)
Determination of light ray path in a prism

3) Extend dc to meet extended ab .The angle between them is the angle of deviation $\alpha$.

4) Measure the angle of incidence $\phi_1$, the angle of refraction $\theta_1$, the inner incidence angle $\phi_2$, the angle of emergence $\theta_2$ and the angle of deviation ($\alpha$).

5) Repeat the previous steps several times changing the angle of incidence and tabulate the results.

| Angle of the prism A | angle of incidence $\phi_1$ | angle of refraction $\theta_1$ | Angle of inner incidence $\phi_2$ | angle of emergence $\theta_2$ | Angle of deviation $\alpha$ |
|---|---|---|---|---|---|
| | | | | | |

Find the minimum angle of deviation and the corresponding angles $\phi_o$ and $\theta_o$ .

- Then obtain the refractive index from equation (2-10).

## The dispersion of light by a triangular prism:

It has been proven previously that in the case of minimum deviation, the refractive index may be determined from the relation:

$$n = \frac{\sin\left(\frac{\alpha_o + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where (n) is the refractive index,$\alpha_0$ is minimum angle of deviation, and (A) is the angle of the prism.

Since the angle of the prism is constant for a certain prism, so the minimum angle of deviation changes by changing the refractive index. As the refractive index increases, the minimum angle of deviation increases and vice versa.



Fig (2-17)
A prism disperses the spectrum

Note also that the refractive index (n) depends on up the wavelength λ, then the minimum angle of deviation depends also on the wavelength. Thus, if a beam of white light falls on a prism set at the minimum angle of deviation, then the emerging light disperses into spectral colors as illustrated in (Fig 2-17).From this figure, it is concluded that the violet ray has the largest deviation ( maximum refractive index). The visible spectral colors into which the white light is dispresed are arranged by the order: red, orange, yellow, green, blue, indigo and violet.

**Go Further**

For more knowledge about this topic you can refer to the Egyptian Knowledge Bank (EKB) through the opposite link



## The thin prism:

A thin prism is a triangular glass prism. Its apex angle is a few degrees and is in the position of minimum deviation:

$$n = \frac{\sin\left(\frac{\alpha_0 + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Since: $\left(\frac{\alpha_0 + A}{2}\right)$ and $\left(\frac{A}{2}\right)$ are small angles.

Thus,

$$\sin\left(\frac{\alpha_0 + A}{2}\right) \cong \left(\frac{\alpha_0 + A}{2}\right) \quad \text{(radians)}$$

and $$\sin\left(\frac{A}{2}\right) \cong \frac{A}{2} \quad \text{(radians)}$$

Substituting from (2-10), we find that the refractive index of the material of the thin prism is determined by

$$n = \frac{\alpha_0 + A}{A} \qquad (2 - 11)$$

$$\alpha_0 = A(n - 1) \qquad (2 - 12)$$

## Dispersive Power

When white light falls on a prism, the light disperses into its spectrum due to the variation of the refractive index with wavelength.

$$(\alpha_o)_r = A(n_r - 1)$$

$$(\alpha_o)_b = A(n_b - 1)$$

where $n_r$ is the refractive index for red and $n_b$ for blue.

Subtracting,

$$(\alpha_o)_b - (\alpha_o)_r = A(n_b - n_r) \qquad (2-13)$$

The LHS represents the angular dispersion between blue and red. For yellow (middle between blue and red), the angle is :

$$(\alpha_o)_y = A(n_y - 1) \qquad (2-14)$$

where $n_y$ is the refractive index for yellow. If $(\alpha_o)_y$ is the average of $(\alpha_o)_r$ and $(\alpha_o)_b$, then $n_y$is the average of $n_r$ and $n_b$. We define$\omega_\alpha$ as :

$$\omega_\alpha = \frac{(\alpha_o)_b - (\alpha_o)_r}{(\alpha_o)_y} = \frac{n_b - n_r}{n_y - 1} \qquad (2-14)$$

where $\omega_\alpha$ is the dispersive power, and is independent of the apex angle.

## In a Nutshell

**•Laws of reflection of light :**

1) Angle of incidence = Angle of reflection
2) The incident ray, the reflected ray, and the normal to the reflecting surface at the point of incidence, all lie in one plane perpendicular to the reflecting surface.

•Light refracts between two media because of the different velocities of light in the two media $v_1$ & $v_2$

**•Laws of refraction of light :**

1) The ratio between the sine of the angle of incidence in the first medium, to the sine of the angle of refraction in the second medium is constant, and is known as the refractive index ${}_1n_2$

$${}_1n_2 = \frac{\sin \phi}{\sin \theta}$$

where $\phi$ is the angle of incidence in the first medium, and $\theta$ is the angle of refraction in the second medium

2) The incident ray, the refracted ray, and the normal to the surface of separation at the point of incidence, all lie in one plane normal to the surface of separation.

• The relative refractive index between two media is the ratio between the velocity of light in the first medium $v_1$ and the velocity of light in the second medium $v_2$

$${}_1n_2 = \frac{v_1}{v_2}$$

• The absolute refractive index for a medium is given by :

$$n = \frac{c}{v}$$

where c is the velocity of light in free space and v is the velocity of light in the medium.

• **Snell's law** :

$$n_1 \sin\phi = n_2 \sin\theta$$

- The distance between two successive similar fringes (either bright or dark) is :

$$\Delta y = \frac{\lambda R}{d}$$

where $\lambda$ is the wavelength of light used R is the distance between the double slit and the screen, and d is the distance between the two slits.

- Light is a wave motion.
- The critical angle is the angle of incidence in the more dense medium, corresponding to an angle of refraction in the less dense medium equal to 90°.
- The absolute refractive index is equal to the reciprocal of the sine of the critical angle when light travels from this medium into air or vacuum.

$$n = \frac{1}{\sin \phi_c}$$

- Total internal reflection takes place when the angle of incidence in the more dense medium is greater than the critical angle.
- The mirage is a phenomenon that can be explained by total internal reflection.
- The angle of the apex of the prism is given by:

$$A = \theta_1 + \phi_2$$

- The angle of deviation is given by:

$$\alpha = (\phi_1 + \theta_2) - A$$

where $\phi_1$ is the angle of incidence $\theta_2$ is the angle of emergence

- In the case of minimum deviation :

$$\phi_1 = \theta_2 = \phi_0$$

$$\theta_1 = \phi_2 = \theta_0$$

- Refractive index of the prism material is given by:

$$n = \frac{\sin\left(\frac{\alpha_o + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where n is the refractive index, $\alpha_o$ is the minimum angle of deviation.

- The minimum angle of deviation in a thin prism is :

$$\alpha_o = A(n - 1)$$

- The angular dispersion for a thin prism is :

$$(\alpha_o)_b - (\alpha_o)_r = A(n_b - n_r)$$

- where $(\alpha_o)_b$ is the minimum deviation angle of the blue ray, and $(\alpha_o)_r$ is the minimum deviation angle of the red ray.
- The dispersive power :

$$\omega_\alpha = \frac{(\alpha_o)_b - (\alpha_o)_r}{(\alpha_o)_y}$$

$$\omega_\alpha = \frac{n_b - n_r}{n_y - 1}$$

where $(\alpha_o)_y$ is the minimum angle of deviation of the yellow light, and $n_y$ is the refractive index for the yellow light.

## Questions and Drills

### I) Essay questions

1) Explain why light is considered to be a wave motion .
2) Describe an experiment to demonstrate the interference of light.
3) Explain how mirage is formed.

### II) Define :

a) the relative refractive index between two media.
b) the absolute refractive index for a medium.
c) the critical angle.
d) the angle of deviation.

### III) Complete :

a) The distance between two successive bright fringes is given by ..................................
b) Snell's law states that : .....................................................
c) The angle of deviation in a thin prism is given from relation :....................
d) The dispersive power is: .....................................................

### IV) Choose the right answer :

1) When light reflects :
   a) the angle of incidence is less than the angle of reflection.
   b) the angle of incidence is greater than the angle of reflection.
   c) the angle of incidence is equal to the angle of reflection.
   d) there is no right answer above .

2) When light refracts, the ratio $\frac{\sin \phi}{\sin \theta}$ ,where $\phi$ is the angle of incidence and $\theta$ is the angle of refraction is:
   a) constant for the two media.
   b) variable for the two media.

c) constant, greater than one.

d) constant, less than one.

3) The ratio between the sine of the angle of incidence in the first medium to the sine of the angle of refraction in the second medium is known as :

a) the absolute refractive index for the first medium.

b) the absolute refractive index for the second medium.

c) the relative refractive index from the second medium to the first medium.

d) the relative refractive index from the first medium to the second medium.

4) The refractive index ${}_1n_2$ is equal to:

a) $\frac{n_2}{n_1}$ b) $\frac{n_1}{n_2}$

c) $n_1 n_2$ d) $\frac{\sin\theta_2}{\sin\theta_1}$

5) The refractive index for the material of a prism in the minimum deviation position is:

a) $n = \frac{\sin\alpha_0}{\sin A}$ b) $n = \frac{\sin\left(\frac{\alpha_0 + A}{2}\right)}{\sin\left(\frac{\alpha_0}{2}\right)}$

c) $n = \frac{\sin\left(\frac{\alpha_0 + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$ d) $n = \frac{\sin\left(\frac{\alpha_0 + A}{2}\right)}{\sin A}$

6) The minimum deviation angle in a thin prism is:

a) $\alpha_0 = n(A-1)$ b) $\alpha_0 = A(n+1)$

c) $\alpha_0 = n(A+1)$ d) $\alpha_0 = A(n-1)$

7) The angle of incidence in a medium is $60^\circ$ and the angle of refraction in the second medium is $30^\circ$. Then the relative refractive index from the first to the second medium is :

a) $\sqrt{3}$　　b) $\sqrt{2}$　　c) $\frac{1}{2}$　　d) 2

8) An incident ray at an angle $48.5^\circ$ on one of the faces of a glass rectangular block ($n = 1.5$), the angle of refraction is :

a) $20^\circ$　　b) $30^\circ$　　c) $35^\circ$　　d) $40^\circ$

9) In an experiment it was found that the minimum angle of deviation is $48.2^\circ$ Given that the angle of the prism is $58.8^\circ$, the refractive index of the material of the prism is :

a) 1.5　　b) 1.63　　c) 1.85　　d) 1/1.85

10) If the critical angle for a medium to air is $45^\circ$, then the absolute refractive index is :

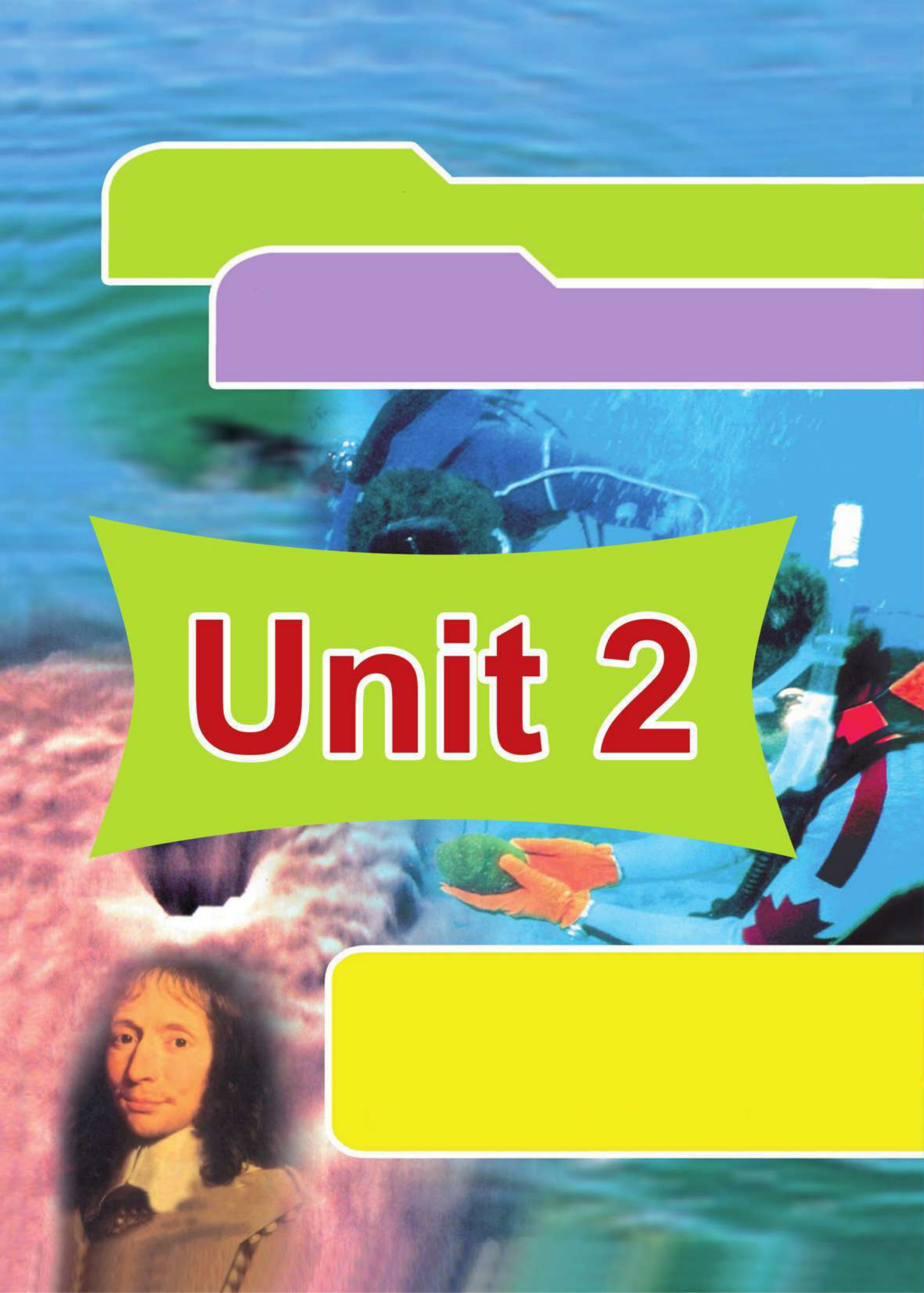
a) 1.64　　b) 2　　c) 1.7　　d) $\sqrt{2}$

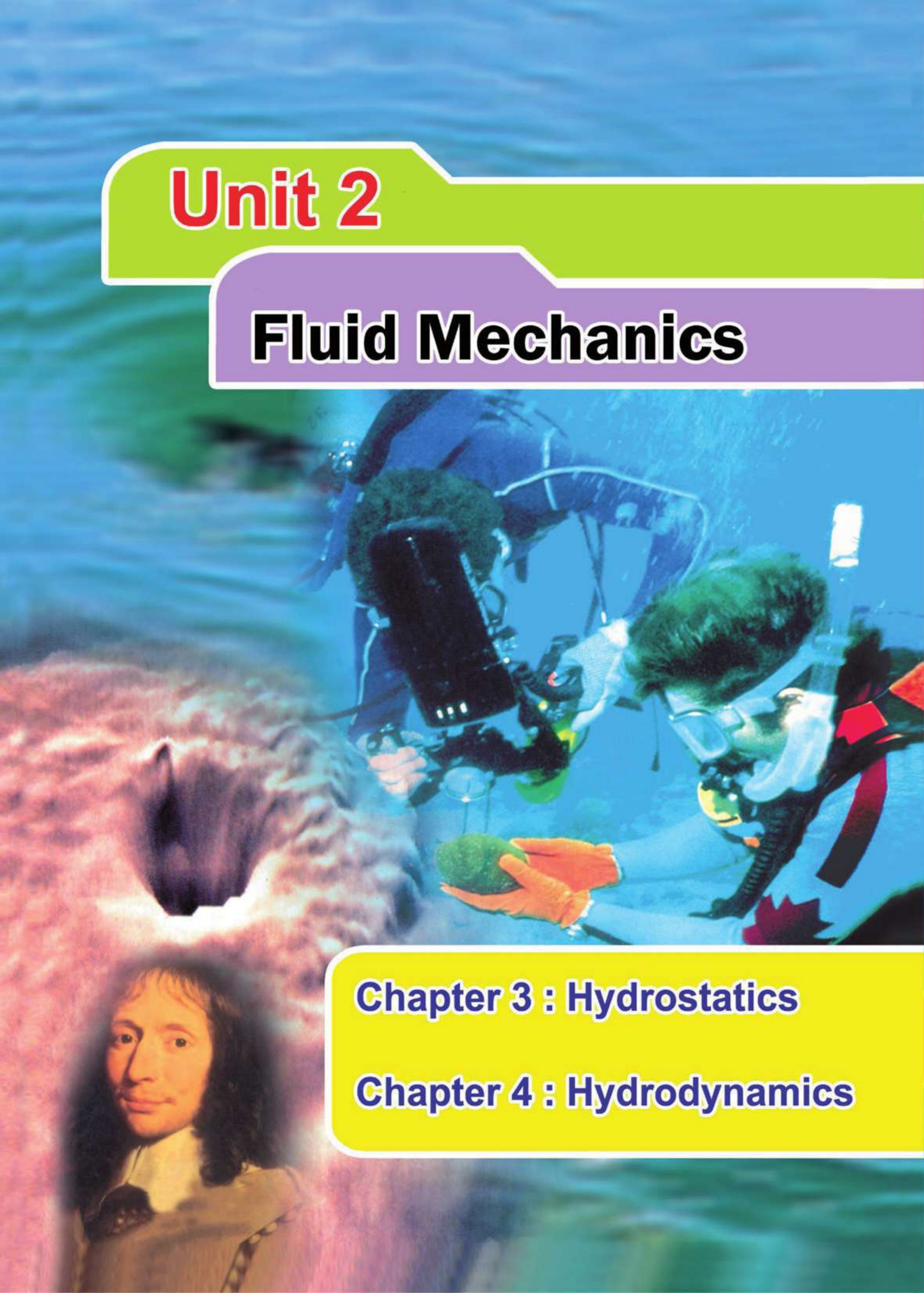
11) A thin prism has an angle of $5^\circ$. Its refractive index is 1.6. It produces a minimum angle of deviation equal to :

a) $5^\circ$　　b) $6^\circ$　　c) $8^\circ$　　d) $3^\circ$

12) A ray of light falls on a thin prism at an angle of deviation $4^\circ$ and its apex angle $8^\circ$.Its refractive index is :

a) 1.5　　b) 1.4　　c) 1.33　　d) 1.6


Unit 2


Unit 2
Fluid Mechanics
Chapter 3 : Hydrostatics
Chapter 4 : Hydrodynamics

# Fluid Mechanics

Unit 2



## Chapter 3 : Hydrostatics

# Chapter 3 Hydrostatics

## Overview

Fluids are materials which can flow. They are liquids and gases. Gases differ from liquids in compressibility. Gases are compressible, while liquids are incompressible. Thus, liquids occupy a certain volume, while gases can fill any volume they occupy, i.e., the volume of the container.

## Density

Density is a basic property of matter. It is the mass per unit volume ($kg / m^3$)

$$\rho = \frac{m}{V_{ol}} \qquad (3\text{-}1)$$

where $V_{ol}$ is the volume

Density varies from one element to another due to:

1) difference in atomic weights

2) difference in interatomic or intermolecular distances or molecular spacings.

We know that bodies of less density float over more dense liquids. The following table shows density for different material.

| Material | Density $kg/m^3$ | Material | Density $kg/m^3$ |
|---|---|---|---|
| Solids: | | Kerosene | 820 |
| Aluminum† | 2700 | Mercury | 13600 |
| Brass | 8600 | Glycerin | 1260 |
| Coper | 8890 | Water | 1000 |
| Glass | 2600 | | |
| Gold | 19300 | Gases: | |
| Ice | 910 | Air | 1.29 |
| Iron | 7900 | Ammonia | 0.76 |
| Lead | 11400 | Carbon dioxide | 1.96 |
| Platinum | 21400 | Carbon mono oxide | 1.25 |
| Steel | 7830 | Helium | 0.18 |
| Suger | 1600 | Hydrogen | 0.090 |
| Wax | 1800 | Nitrogen | 1.25 |
| | | Oxygen | 1.43 |
| Liquids: | | | |
| Ethyl Alchol | 790 | | |
| Benzene | 900 | | |
| Blood | 1040 | | |
| Gasoline | 690 | | |

The ratio of density of any material to that of water at the same temperature is called the relative density of the material (no units). The relative density of a material, is equal to :

$$= \frac{\text{the density of the material at a certain temperature}}{\text{the density of water at the same temperature}} \qquad (3 - 2)$$

$$= \frac{\text{the mass of a certain volume of matter at a certain temperature}}{\text{the mass of the same volume of water at the same temperature}}$$

## Applications to density

1)Measuring density is of great importance in analysis, such as measuring the density of the electrolyte in a car battery. When the battery is discharged, the density of the electrolyte (dilute sulfuric acid) is low due to chemical reaction with the lead plates and the formation of lead sulfate. When the battery is recharged, the sulfate is loosened from the lead plates and go back to the electrolyte, and the density increases once more. Thus, measuring the density indicates how well the battery is charged.

2) Measuring density is used in clinical medicine, such as measuring blood and urine densities. Normal blood density is 1040 kg / m$^3$ – 1060 kg / m$^3$. High density indicates higher concentrations of blood cells and lower concentrations indicate anemia.

The normal urine density is 1020 kg / m$^3$. In some diseases, salts increase and cause the urine density to increase.

**Go Further**

For more knowledge about this topic you can refer to the Egyptian Knowledge Bank (EKB) through the opposite link



## Pressure

Pressure at a point is the average force which acts normal to unit area at that point. If force F is normal to a surface of an area A, then the affected pressure P on the surface is determined by the following relation:

$$P = \frac{F}{A} \qquad (3-3)$$

* Since The Force measured in Newton (N) And The Area measured in (m$^2$), The measuring unit which the pressure is measured is (N/m$^2$)

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### *Elephant's foot vs human foot*

Because the pressure is the force per unit area, the pressure due to a pointed high heel is greater than the pressure due to an elephant's foot, since the area of the pointed heel is very small (Fig 3 – 1).

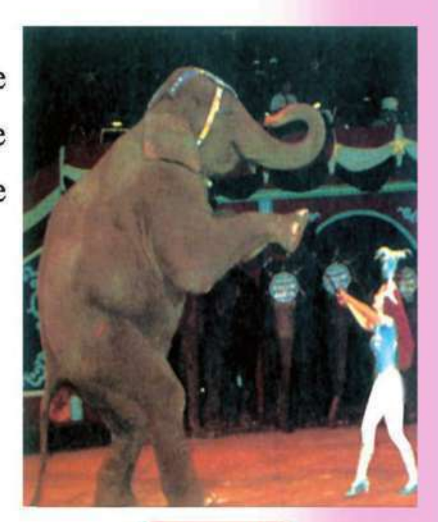
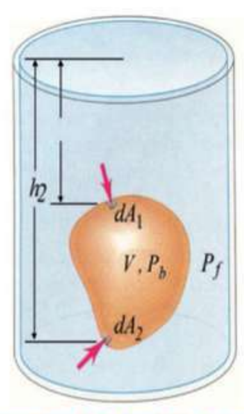


Fig (3–1)
Meaning of pressure

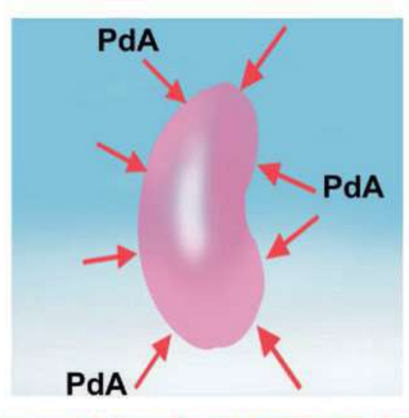
## Pressure at a point inside a liquid and its measurement.

If you push a piece of foam under water and let it go, it will rise and float. This indicates that water pushes the immersed foam by an upward force. This force is due to the pressure difference across this piece of foam.

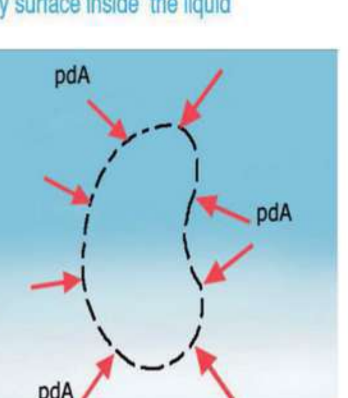
At any point inside a liquid, the pressure acts in any direction. The direction of the force due to the pressure on any surface is normal to that surface. The pressure on a body is the same as the pressure on a volume of the liquid that has the same shape of the body in case this body were removed. In other words, the liquid occupying the same size which a body would occupy is



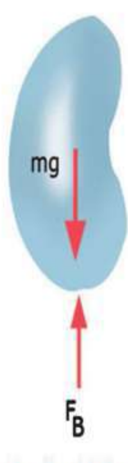

a) pressure inside a liquid is perpendicular to any surface inside the liquid




b) pressure is perpendicular to the surface of an immersed body at every point




c) pressure on the surface of an object is equal to the pressure on the surface of a similar size of the liquid of the same volume and shape




d) in a certain size of a liquid there is equilibrium between two forces: the weight of the liquid and the pressure due to the remainder of the liquid.

Fig (3 – 2)

Pressure in a liquid

acted upon by two forces: its weight downwards and the force due to the pressure of the liquid around it. As the depth of the liquid increases, the pressure increases (Fig 3 – 3).

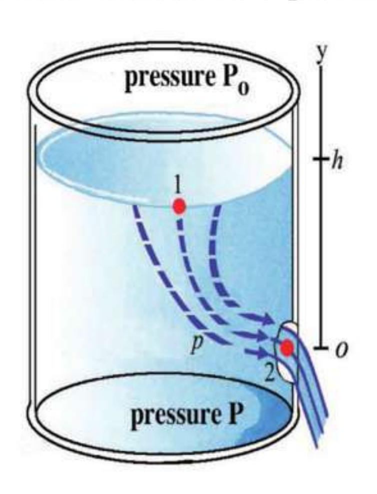





Fig (3 – 3)

Pressure increases with liquid depth

To calculate the pressure (P), we imagine a horizontal plate x of area A at depth h inside a liquid of density $\rho$ (Fig 3 – 4). This plate acts as the base of a column of the liquid. The force acting on the plate x is the weight of the column of the liquid whose height is h and whose cross section is A.

Because the liquid is incompressible, the force resulting from the liquid pressure must balance with the weight of the column of the liquid. The volume of this column is Ah and its mass Ah$\rho$, hence its weight $F_g$ is given by :

$$F_g = Ah\rho g$$

Fig (3 – 4)

Calculation of the pressure of a column of liquid

where g the acceleration due to gravity. The pressure due to the liquid from under the plate x (acting upwards) must be :

$$P = \frac{F}{A} = \frac{Ah\rho g}{A}$$

$$\therefore P = h\rho g \qquad (3-4)$$

Taking into consideration the fact that the free surface of the liquid is subject to atmospheric pressure $P_a$ ,then the total pressure at a point inside a liquid it depth d is given by:

$$\boxed{P = P_a + h\rho g} \qquad (3-5)$$

Practical observations show indeed that the liquid pressure at a point inside it increases with increasing depth and with increasing density at the same depth.

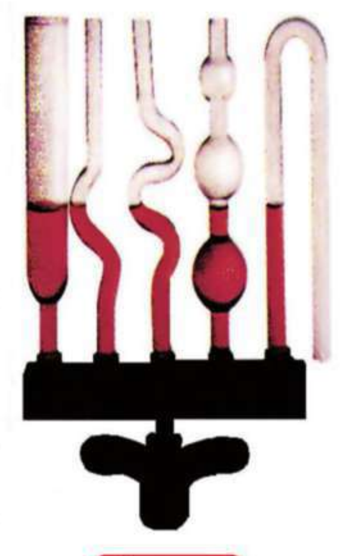


Fig (3 – 5)

Water rises to the same level in connecting vessels

Thus, we conclude :

1) All points that lie on a horizontal plane inside a liquid has the same pressure.
2) The liquid that fills connecting vessels rise in these vessels to the same height, regardless of the geometrical shape of these vessels provided that the base is in a horizontal plane (Fig 3 – 5).

   Therefore, the average sea level is constant for all connected seas and oceans.
3) The base of a dam must be thicker than that the top to withstand the increasing pressure at increasing depths (Fig 3 – 6).

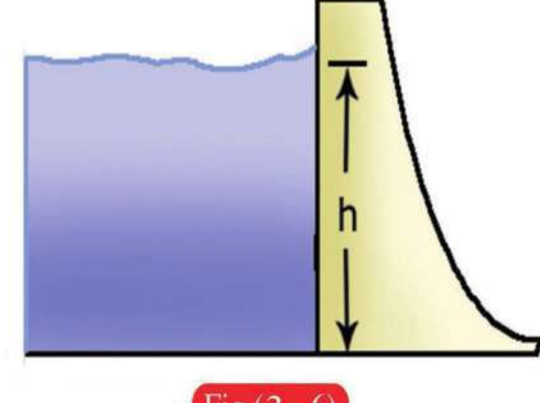



Fig (3 – 6)

Dams must be thicker at the base to withstand the pressure at increasing depths

## Balance of liquids in a U - shaped tube

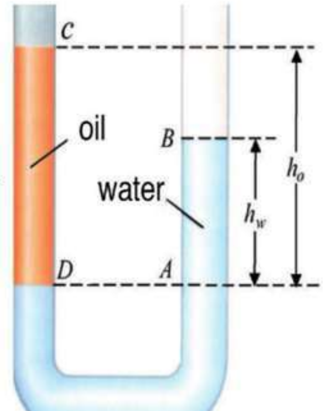
Let us take a U - shaped tube filled with an appropriate amount of water. Let us add a quantity of oil in the left branch of the tube, until the level of oil reaches level C at a height $h_o$ over the separating surface AD between water and oil, noting that both liquids do not mix. Let the height of the water in the right branch be $h_w$ above level AD (Fig 3 – 7).

Because the pressure at A = pressure at D

$$\therefore P_a + \rho_o g h_o = P_a + \rho_w g h_w$$

where $P_a$ is the atmospheric pressure, $\rho_o$ the density of oil, $\rho_w$ the density of water. Thus, $h_o \rho_o = h_w \rho_w$ or

$$\frac{\rho_o}{\rho_w} = \frac{h_w}{h_o} \qquad (3\text{-}6)$$




Fig(3 – 7)
Balance of liquids in a U - shaped tube

Measuring $h_o$ and $h_w$, we may determine practically the density of oil, knowing the density of water.

## Atmospheric Pressure

Torcelli invented the mercury barometer to measure the atmospheric pressure. He took a 1 m long glass tube and filled it completely with mercury and turned it upside down in a tank of mercury. He noticed that the level of mercury went down to a certain level that measured 0.76 m from the surface of mercury in the tank. The void above the column of mercury in the tube is vacuum (neglecting mercury vapor) is called Torcelli vacuum.

From Fig (3 – 8), the height h of the mercury column in the tube is constant, wether the tube is upright or inclined. Taking two points A, B in one horizontal plane (Fig 3 – 9), such that A is outside the tube at the surface of mercury in the tank, while B is inside the tube. The pressure at B = the pressure at A. Thus:

$$\boxed{P_a = \rho g h} \qquad (3 - 7)$$

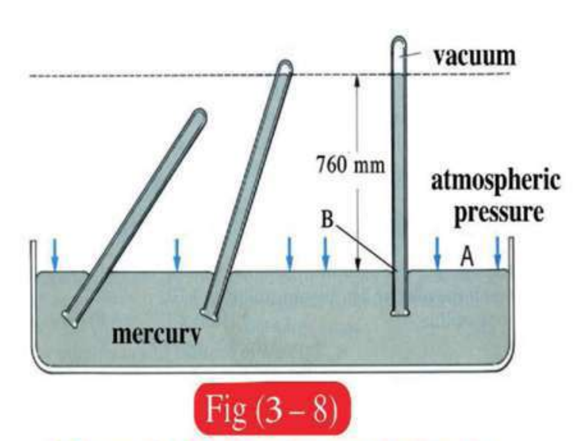
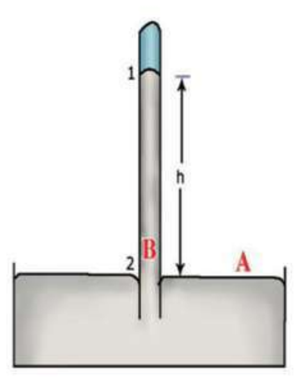



Fig (3 – 8)

Mercury height in a barometer is not affected by the tilting of the manometer

This means that the atmospheric pressure is equivalent to the weight of a column of mercury whose height is 0.76 m and cross sectional area $1m^2$ at OC° at sea level. This is known as S.T.P. (standard temperature and pressure). Since the density of mercury at O C° is 13595 $kg/m^3$ and g = 9.8 $m/s^2$




$$P_a = 1 \text{ Atm} = 0.76 \times 13595 \times 9.81$$

$$= 1.013 \times 10^5 \text{ N/m}^2$$

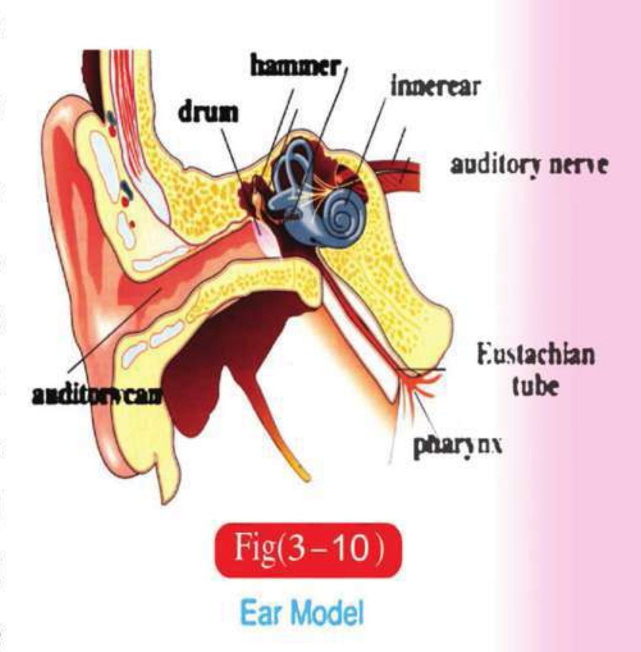
Fig (3 – 9)

A simple barometer

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### What happens to the ear at heights

The atmospheric pressure is the weight of a column of air above the Earth's surface per unit area. As we go up, the height of this column decreases, so does the pressre. At the eardrum, the external pressure must be balanced out by an internal pressure. When the external pressure decreases, we feel tense at the eardrum, since the internal pressure pushes it outwards. This can be compensated by adjusting the amount of air in the Eustachian tube by swallowing and chewing gum to reduce the pressure on the eardrum (Fig 3 – 10).




Fig(3 – 10)

Ear Model

## Units for measuring atmosphere pressure

From the previous relation, it is clear that the units which the atmosphelic pressure is measured in international system ( S.I ) is $N/m^2$ .

Pressure has units $N/m^2$. This is called Pascal.

1 Pascal = $N/m^2$

Thus, the atmospheric pressure $P_a$ is given by.

$P_a = 1.013 \times 10^5$ Pascal

Defining ($10^5 N/m^2$) as Bar = $10^5$ Pascal

$P_a = 1.013$ Bar

We also use Torr unit .

$$1 \text{ Torr} = 1 \text{ mm Hg}$$

$$P_a \text{ (1 Atm)} = 760 \text{ Torr} = 760 \text{ mm Hg} = 0.76 \text{ m Hg} = 1.013 \text{ Bar}$$

## Manometer

The manometer is a U - shaped tube containing a proper amount of liquid of a known density. One end is connected to a gas reservoir. The level of the liquid in the manometer may rise in one branch and go down in the other. Taking two points A,B in one horizontal plane in the same liquid (Fig 3 – 11 a), we have the pressure at B = the pressure A

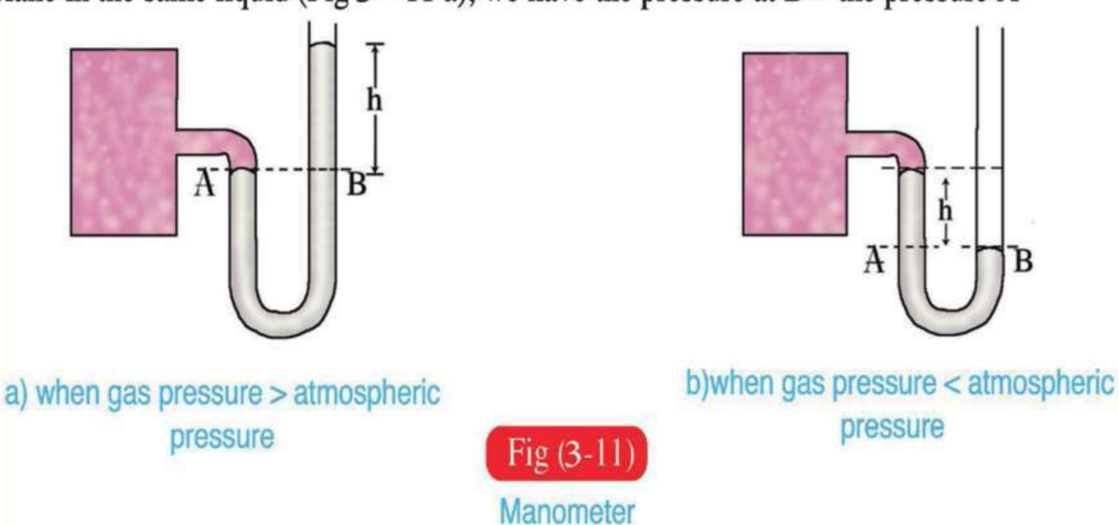



Fig (3-11)
Manometer

When P- the pressure of the gas enclosed in the reservoir - is greater than $P_a$, $\rho gh$ is the weight of a column the liquid in the free end of the manometer above point B and is the difference between P and $P_a$ (Fig 3 -11a),

$$P = P_a + \rho gh$$

In the case $P < P_a$ (Fig 3 – 11 b),

$$P = P_a - \rho gh$$

i.e., the level of the liquid in the free end branch is lower than the level of the liquid in the end connected to the gas reservoir by a height h. In many cases, it suffices to measure the pressure difference,

$$\Delta P = P - P_a = \rho gh \qquad (3-8)$$

Knowing the liquid density $\rho$ in the manometer and the height difference h between the liquid levels in the two branches and the acceleration due to gravity g,we can calculate $\Delta P$. Knowing $P_a$ ,we may determine P of the gas enclosed in the reservoir.

## Applications to Pressure

1) Blood is a viscous liquid pumped through a complicated network of arteries and veins by the muscular effect of the heart. This is called steady flow (chapter 4). In the case of turbulent flow (chapter 4), there is noise which can be detected by a stethoscope. There are two values for blood pressure: the systolic pressure, as blood pressure is maximum (normally 120 Torr). This occurs when the cardiac muscle contracts and blood is pushed from the left ventricle to the aorta onto the arteries. The diastolic pressure is the minimum blood pressure (normally 80 Torr) when the cardiac muscle relaxes.
2) When a tire is well inflated (under high pressure) the area of contact with the road is as small as possible, while an underinflated (low pressure) tire has large contact area. As the area of contact with the road increases, friction increases and consequently, the tire is heated. Air pressure in a tire can be measured by a pressure gauge (Fig 3 – 12).

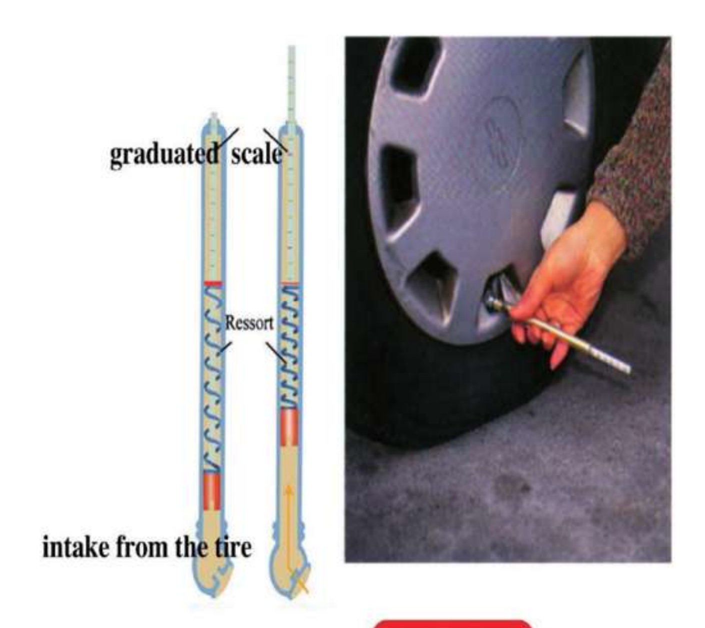



Fig (3-12)

Measuring tire pressure with a pressure gauge

## Examples

1) A solid parallelepiped (5cm x 10cmx 20cm) has density 5000kg /m$^3$ is placed on a horizontal plane. Calculate the highest and lowest pressure. (g = 10 m/s$^2$)

### Solution

For the highest pressure it is placed on the side with the least area (5 cm x 10 cm), where the force is the weight.

$$P = \frac{F_g}{A} = \frac{5 \times 10 \times 20 \times 10^{-6} \times 5000 \times 10}{5 \times 10 \times 10^{-4}} = 10^4 \ N/m^2$$

For the lowest pressure, it is placed on the side of the greatest area (10 cm x 20cm)

$$P = \frac{F_g}{A} = \frac{5 \times 10 \times 20 \times 10^{-6} \times 5000 \times 10}{10 \times 20 \times 10^{-4}} = 2500 \ N/m^2$$

2) Find the total pressure and the total force acting on the base of a tank filled with salty water of density 1030 kg/m$^3$. If the cross-section of the base is 1000 cm$^2$ , the height of the water is 1cm and the surface of the water is exposed to air. Take g = 10 m/s$^2$ and the atmospheric pressure = 1 Atm = 1.013 x 10$^5$N/m$^2$

### Solution

Total pressure

$$P = P_a + \rho g h$$

$$= 1.013 \times 10^5 + 1030 \times 10 \times 1$$

$$= (1.013 + 0.103) \times 10^5 = 1.116 \times 10^5 \ N/m^2$$

Total force

$$F = P \times A = 1.116 \times 10^5 \times 1000 \times 10^{-4}$$

$$= 1.116 \times 10^4 \ N$$

3) A mercury manometer is used to measure the pressure of gas in a reservoir. The mercury level in the free end is higher than the mercury level in the side connected to the reservoir by 36 cm. What is the pressure of the trapped gas in :

a) cm Hg units    b) Atm units    c) $N/m^2$ units

Take the atmospheric presssure to be 0.76 m Hg = $1.013 \times 10^5$ $N/m^2$

**Solution**

a) In cm Hg units

$$P = 76 + 36 = 112 \text{ cm Hg}$$

b) In Atm units

$$P = \frac{P \text{ cm. Hg}}{76} = \frac{112}{76} = 1.474 \text{ Atm}$$

c) In $N/m^2$ units

$$P = 1.474 \times 1.013 \times 10^5 = 1.493 \times 10^5 \text{ N/m}^2$$

4) A U- shaped tube of cross sectional area on the narrow side of 1 $cm^2$ and on the wide side $2cm^2$, is partially filled with water (density 1000 $kg/m^3$). A quantity of oil (density 800 $kg/m^3$) is poured into the narrow side until of the height of the oil column reaches 5 cm. Calculate the height of water above the surface of separation.

**Solution**

$$P = \rho_o g\, h_o = \rho_w g\, h_w$$

$$\therefore \rho_o h_o = \rho_w h_w$$

$$h_w = \frac{\rho_o h_o}{\rho_w} = \frac{800 \times 5}{1000} = 4 \text{ cm}$$

**Note:**

The radius of the tube (or the cross sectional area) has no effect at all on the height of each liquid in both branches of the tube. Hence, the liquid level is not influenced by the shape of the tube.

## Pascal's principle

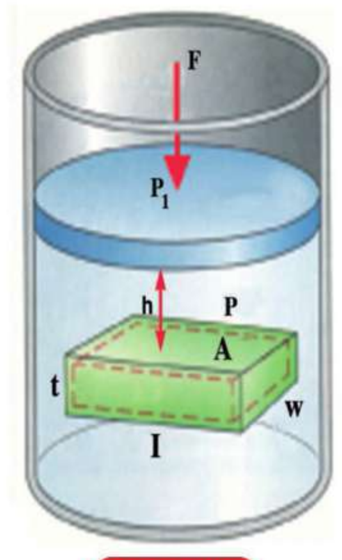


Fig (3-13)
Increase of weight on the piston increases the pressure in the liquid

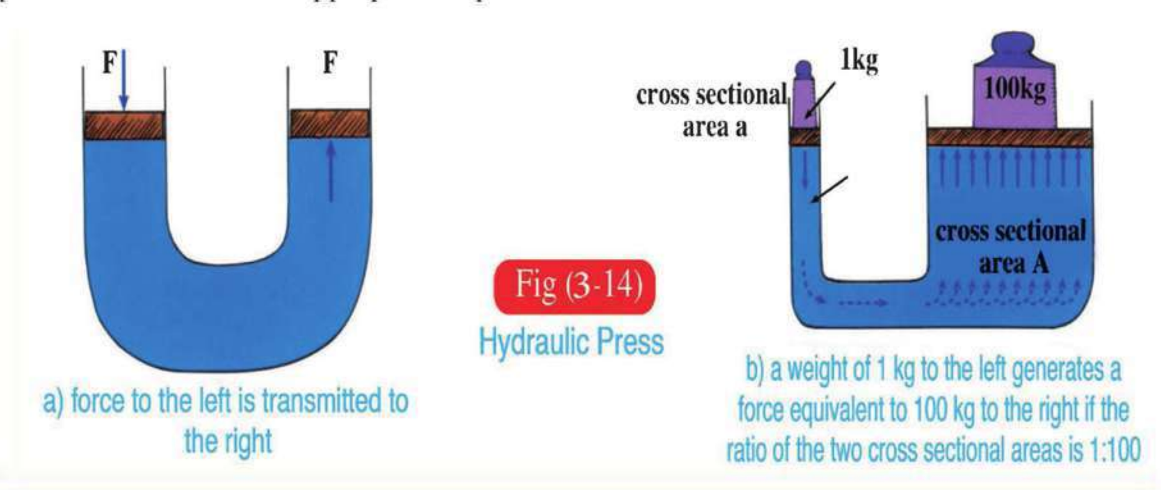
Consider a glass container (Fig 3 – 13) partially filled with liquid and equipped with a piston at the top. The pressure at a point A inside the liquid at depth h is $P = P_1 + h\rho g$ where $P_1$ is the pressure immediately underneath the piston, which results from the atmospheric pressure, as well as the weight of the piston and the force applied on the piston. If we increase the pressure on the piston by an amount $\Delta P$, by placing an additional weight on the piston. We note that the piston does not move inside because the liquid is incompressible. But the pressure underneath the piston must increase in turn by $\Delta P$. This raises the pressure at point A by $\Delta P$. This make the pressure $P = P_1 + \rho g h + \Delta P$.

Pascal formulated this result as follows :

**When pressure is applied on a liquid enclosed in a container, the pressure is transmitted in full to all parts of the liquid as well as to the walls of the container.** This is known as Pascal's principle or Pascal's rule.

## Application to Pascal's rule : hydraulic Press

The hydraulic press (Fig 3 – 14) consists of a small piston whose cross sectional area is "a" and a large piston whose cross sectional area is "A". The space between the two pistons is filled with an appropriate liquid.



a) force to the left is transmitted to the right

Fig (3-14)
Hydraulic Press

b) a weight of 1 kg to the left generates a force equivalent to 100 kg to the right if the ratio of the two cross sectional areas is 1:100


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If pressure P is exerted to the small piston, this pressure is transmitted to the liquid and, subsequently, to the surface of the large piston. If the force applied to the small piston is f and the force affecting the large piston is F, and because the pressure on both pistons must be the same at equilibrium at the same horizontal plane, then :

$$P = \frac{f}{a} = \frac{F}{A}$$

$$F = \frac{A}{a} f \qquad (3 - 9)$$

From this relation, it is clear that if force f affects a small piston, a large force F is generated on the large piston. The mechanical advantage of the hydraulic press $\eta$ is given by:

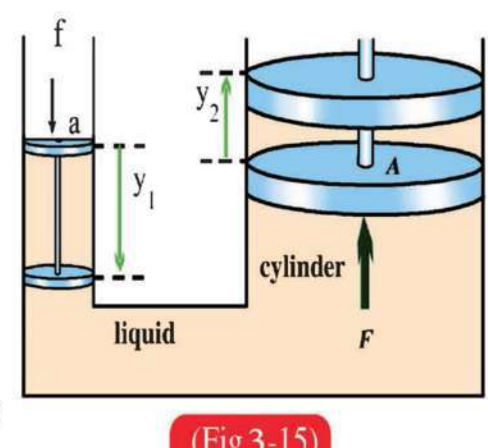
$$\eta = \frac{F}{f} = \frac{A}{a} \qquad (3 - 10)$$

Thus, the mechanical advantage of a hydraulic press is determined by the ratio of the large piston to the small piston. Referring to Fig (3 – 15), it is clear that if the small piston moves down a distance $y_1$ under the influence of f, then the large piston moves up a distance $y_2$ under the effect of F. According to the law of conservation of energy, the work done in both cases must be the same (for 100% piston efficiency),

$$f y_1 = F y_2$$

$$\frac{f}{F} = \frac{y_2}{y_1}$$

$$F = \frac{y_1}{y_2} f \qquad (3 - 11)$$




(Fig 3-15)
Mechanical advantage

This shows that the mechanical advantage of the piston may alternatively be expressed as the ratio $y_1/y_2$

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## *Applications to Pascal's rule*

1) The hydraulic brake in a car uses Pascal's rule as the braking system uses a brake fluid. Upon pushing on the brake pedal with a small force and a relativeley long stroke (distance), the pressure is transmitted in the master brake cylinder, hence, onto the liquid and the whole hydraulic line, then to the piston of the wheel cylinder outwardly, and finally to the brake shoes and the brake drum. A force of friction results, which eventually stops the car. This type of brakes is called drum brake (rear brake) (Fig 3 – 16). In the case of the front (disk) brake (Fig 3 – 17), the forces resulting from the braking action press on the brake pads which produce friction enough to stop the wheel. It should be noted that the distance traveled by the brake shoes in both cases is small because the force is large.

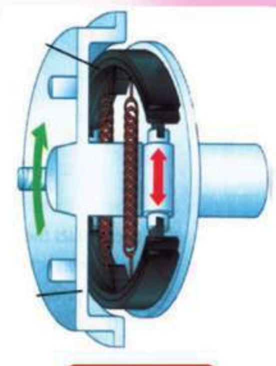
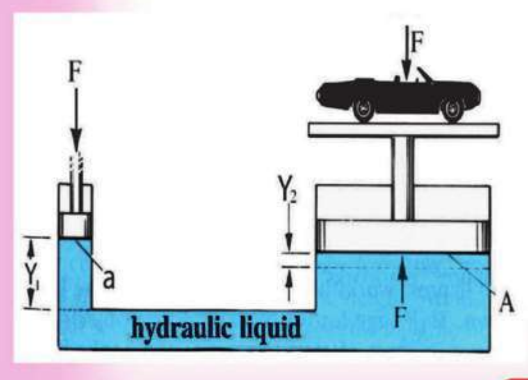


Fig (3-16)
Rear brakes



Fig (3-17)
Front brakes

2) In another application to Pascal's rule, a hydraulic lift uses a liquid to lift up cars in gas stations (Fig 3 – 18).



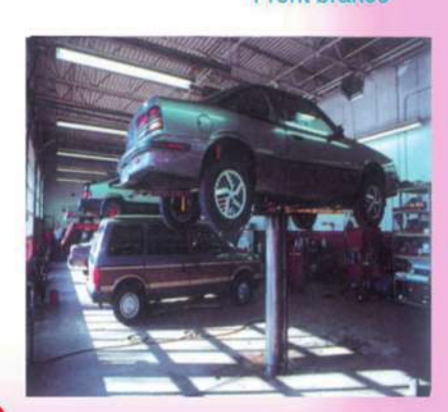



Fig (3-18)
A hydraulic lift

3) The caterpillar also uses Pascal's rule (Fig 3 – 19).

4) A diver wears a diving suit and a helmet to protect him from pressures at large depths. At low (shallow) depths, the diver - without the helmet - blows air in his sinuses to balance the external pressure (Fig 3 – 20). At large depths, the diving suit is appropriately inflated with air, and the helmet protects the diver's head from crushing pressures (Fig 3 – 21).



Fig (3-19)

A hydraulic caterpillar



Fig (3-20)

Diving at low depths



Fig (3-21)

Diving at large depths (500 m)

## Examples

A hydraulic press has cross sectional area $10 cm^2$ which is acted on by a force of 100N. The large cross sectional area is $800 \ cm^2$. Taking $g = 10 m/s^2$ ,calculate :

a) the largest mass that can be lifted by the press

b) the mechanical advantage of the press

c) the distance traveled by the small piston so that the large piston moves a distance of 1cm

**Solution**

The force acting on the large piston :

$$\frac{F}{a} = \frac{F}{A}$$

$$F = \frac{100}{10} \times 800 = 8 \times 10^3 N$$

a)To calculate the largest mass that can be lifted by the large piston,

$$m = \frac{F}{g} = \frac{8 \times 10^3}{10} = 800 \text{ kg}$$

b)To calculate the mechanical advantage,

$$\eta = \frac{F}{f} = \frac{A}{a} = \frac{800}{10} = 80$$

c)To calculate the distance traveled by the small piston,

$$fy_1 = F y_2$$

$$y_1 = \frac{8000 \times 1}{100} = 80 \text{ cm}$$

## In a Nutshell

### I.Definitions and Basic Concepts

- The density ($\rho$) is the mass per unit volume ($kg/m^3$)
- The pressure P at a point is the normal force acting on a unit surface area ($N/m^2$).
- All points lying in the same plane have the same pressure.
- The atmospheric pressure is equivalent to the pressure produced by the weight of a mercury column of height about 0.76 m and base area $1m^2$ at 0°C
- The units of atmospheric pressure are :

  a) Pascal ($1\ N/m^2$).

  b) Bar ($10^5\ N/m^2$).

  c)cm Hg.

  d) Torr (mm. Hg).
- The manometer is an instrument for measuring the difference in the pressure of a gas inside a container and the outer atmospheric pressure.
- Pascal's principle :

**The pressure applied on an enclosed liquid is transmitted undiminished to every portion of the liquid and to the walls of the container.**

## II. Basic Formulas :

- Density $\rho = \frac{\text{mass}}{\text{volume}}$ $\qquad \rho = \frac{m}{V_{ol}}$

- Pressure $P = \frac{\text{Force}}{\text{area}}$ $\qquad P = \frac{F}{A}$

- The pressure at a point inside a liquid of density $\rho$ at depth h below its free surface is given by :

$$P = P_a + \rho g h$$

where $P_a$ is the atmospheric pressure and g is the acceleration due to gravity.

- The balance of liquids in a U- shaped tube yields.

$$\rho_1 h_1 = \rho_2 h_2$$

- From Pascal's principle :

$$\frac{f}{a} = \frac{F}{A}$$

where f is the force acting on the small piston of area (a) and (F) is the force resulting on the large piston of the area A.

- The mechanical advantage $= \frac{F}{f} = \frac{A}{a}$
- In order to measure the distance moved by the small piston $y_1$ use the relation.

$$f y_1 = F y_2$$

## Questions and Drills

**I) Put mark (✓) to the correct statement:**

1) The following factors affect the pressure at the bottom of a vessel except one, tick it:

a) the liquid depth in the vessel.
b) the density of the liquid
c) the acceleration due to gravity.
d) the atmospheric pressure.
e) the area of the vessel base.

2) Which of the following factors have no effect on the height of mercury column in a barometer?

a) the density of mercury
b) the cross sectional area of the tube.
c) atmospheric pressure.
d) the acceleration due to gravity.
e) the temperature of mercury.

3) If the ratio between large and small piston diameters is 9:2. The ratio between the two forces on the two pistons are :

a) 9:2
b) 2: 9
c) 4:18
d) 81 : 4
e) 4:81

## II) Define each of the following :

1. Density
2. Pressure at a point
3. Pascal's principle

## III) Essay questions :

1) Prove that the pressure (P) at depth (h) in a liquid is determined from the relation.

$$P = P_a + \rho g h$$

where $P_a$ is the atmospheric pressure, $\rho$ the liquid density and g is the acceleration due to gravity.

2) Describe the manometer and show how it can be used for measuring a gas pressure inside a container.

3) What is meant by Pascal's principle ? Describe one of its applications.

## IV) Drills :

1) The pressure on the base of a cylinder containing oil with diameter 8 m is $1.5 \times 10^3$ $N/m^2$. Find the total force on the base.

**(75400 N)**

2) A difference in pressure of $3.039 \times 10^5$ N/m$^2$ is recommended for air in a car tire. If the atmospheric pressure is $1.013 \times 10^5$ N/m$^2$ ,calculate the absolute pressure of air in the tire in unit of atmosphere.

(4 Atm)

3) A fish tank of cross-sectional area 1000 cm$^2$ contains water of weight 4000N. Find the pressure on its base.

($0.4 \times 10^5$ N/m$^2$)

4) The large and small piston diameters of a hydraulic press are 24cm and 2cm respectively. Calculate the force that must be applied to the small piston to obtain a force of 2000 N on the large piston. Then calculate the mechanical advantage.

(13.9 N, 144)

5)The atmospheric pressure on the surface of a lake is 1Atm. The pressure at its bottom is 3 Atm. Calculate the depth of the lake (density of water 1000 kg/m$^3$, 1 Atm = $1.013 \times 10^5$ N/m$^2$, g = 9.8 m/s$^2$ ).

(20.673 m)

6) A man carries a mercury barometer with readings 76 cm Hg and 74.15 cm Hg at the lower and upper floors, respectively. Calculate the average density of air between the two floors if mercury density is 13600 kg / m$^3$, the building height is 200m and g = 9.8m/s$^2$.

(1.258 kg/m$^3$)

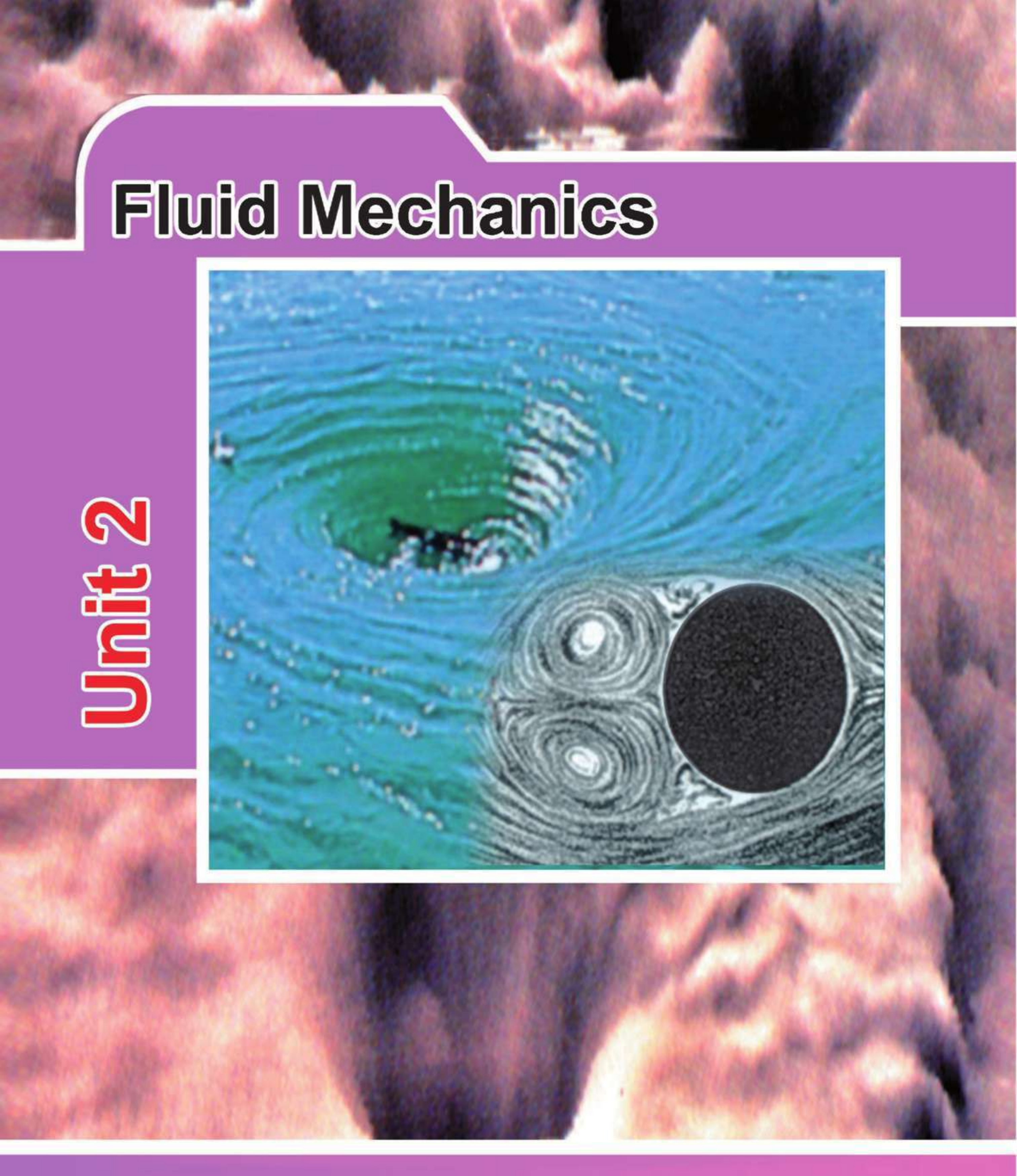
7) A manometer containing mercury is attached to gas enclosed in a container. If the difference of height in the manometer is 25 cm.

Calculate the pressure difference and the absolute pressure of the enclosed air in units of N/m$^2$ (1Atm = $1.013 \times 10^5$ N/m$^2$, mercury density = 13600 kg/m$^3$ and g=9.8 ms$^2$)

($0.3332 \times 10^5$ N/m$^2$, $1.3462 \times 10^5$ N/m$^2$).

# Fluid Mechanics

## Unit 2



## Chapter 4 : Hydrodynamics

## Chapter 4 Hydrodynamics

### Overview

Hydrodynamics (Fluid dynamics) deals, with fluids in motion. We must distinguish between two types of fluid motion, steady flow and turbulent flow.

### Steady flow

If a liquid moves such that its adjacent layers slide with respect to each other smoothly, we describe the motion as a laminar flow or a streamline (steady) flow. In this type of flow, particles of the liquid follow continuous paths called streamlines.Thus, we may visualize the liquid as if it is in a real or virtual tube containing a flux of streamlines representing the paths of the different particles of the liquid (Fig 4 – 1). These streamlines do not intersect, and the tangent at any point along the streamline determines the direction of the instantaneous velocity of each particle of the liquid at that point. The number of streamlines crossing perpendicularly a unit area at a point (density of streamlines) expresses the velocity of flow of the liquid at that point. Therefore, streamlines cram up at points of high velocity and keep apart at points of low velocity.

### Conditions of Steady Flow

1)The rate of flow of the liquid is constant along its path, since the liquid is incompressible and the density of the liquid is independent of distance or time.

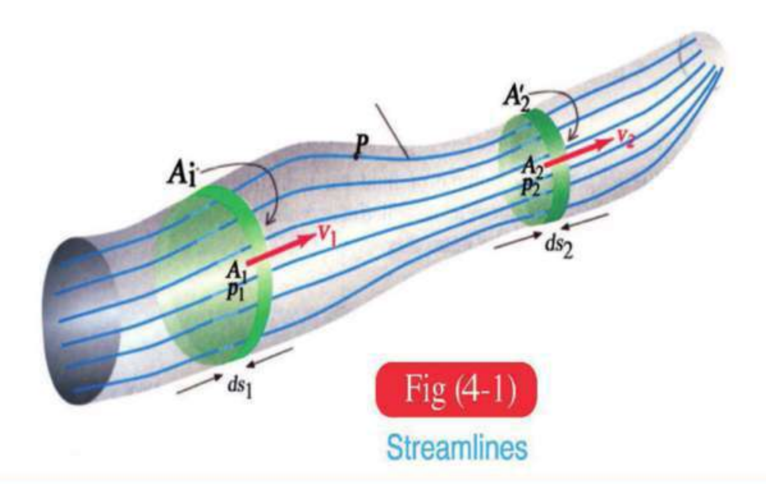



Fig (4-1)
Streamlines

2) In steady flow, the velocity of the liquid at each point is independent of time.

3) The flow is irrotational, i.e., there is no vortex motion.

4)If no forces of friction exist between the layers of the liquid the flow is nonviscous .If there is friction it is viscous.

## Turbulent flow

If the velocity of flow of a liquid exceeds a certain limit, steady flow changes to turbulent flow, which is characterized by the existence of vortices (Fig 4–2). The same thing happens to gases as a result of diffusion from a small space to a large space or from high pressure to low pressure (Fig 4 – 3).



Fig (4-3)
Smoke changes from steady to turbulent flow

## Rate of flow and the continuity equation

We shall focus on steady flow. Consider a flow tube such that:

1) the liquid fills the tube completely.
2) the quantity of the liquid entering the tube at one end equals the quantity of the liquid emerging out from the other end within the same time.
3) the velocity of the liquid flow at any point in the tube does not change with time. There is a relation that ties the rate of flow of the liquid with its velocity and cross sectional area. This relation is called the continuity equation. To understand what the continuity equation entails, we choose



Fig (4-2)
Vortices due to turbulent flow or a violent motion of a body through a liquid

two perpendicular planes normal to the streamlines at the two sections (Fig 4 - 4). The cross sectional area at the first plane is $A_1$ and the cross sectional area at the second plane is $A_2$. The volume rate of flow is the volume of the liquid flowing through area $A_1$ in unit time.We have $Q_v = A_1v_1$, where $v_1$ is the velocity of the liquid at section $A_1$. The mass of the liquid (of density $\rho$) flowing in unit time is called the mass rate of flow $Q_m$ ,which is given by:

$Q_m = \rho Q_v = \rho A_1 v_1$

Similarly, the mass rate of flow through area $A_2$ is $\rho Q_v = \rho A_2 v_2$. Since the mass rate of flow is constant in steady flow

$$\rho A_1 v_1 = \rho A_2 v_2 \qquad (4\text{-}1)$$

$$A_1 v_1 = A_2 v_2$$

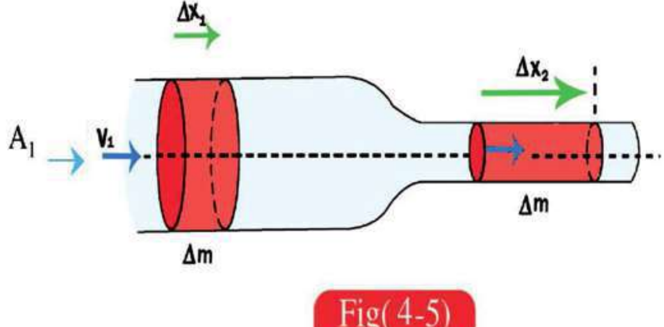
This is the continuity equation leading to

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} \qquad (4\text{-}2)$$

Fig (4-4)

Model for deducing the continuity equation

From this relation, we see that the velocity of the liquid at any point in the tube is inversely proportional to the cross sectional area of the tube at that point. The liquid flows slowly where the cross sectional area $A_1$ is large and flows rapidly, when the cross sectional area $A_2$ is small (Fig 4 – 5). To understand the continuity equation better, we consider a small amount of liquid $\Delta m = \rho \Delta V_{ol}$, where $\Delta V_{ol} = A_1 \Delta x_1$ ,where $\Delta x_1$ is the distance traveled by the liquid in time $\Delta t$ . Thus, $\Delta x_1 = v_1 \Delta t$. Then $\Delta V_{ol} = A_1 v_1 \Delta t$. This same value must emerge from the other side of the tube, since the liquid is incompressible ,i.e, $\Delta V_{ol} = A_2 v_2 \Delta t$ . Thus, we must emphasize that the rate of flow of the liquid is a volume rate $Q_v$ ($m^3/s$), or a mass rate of flow (kg/s). Both of these rates are constant for any cross section. This is called the conservation of mass, which leads to the continuity equation.




Fig( 4-5)

Basis for the continuity equation

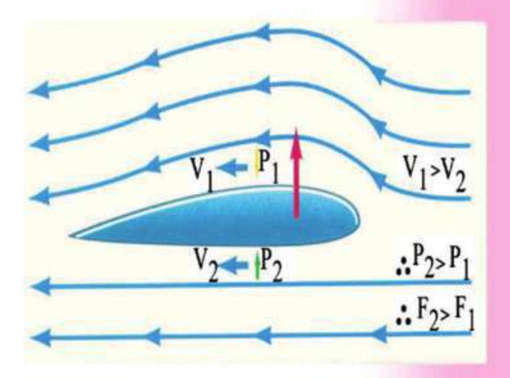
Learn at Leisure

## Why does a plane fly and not fall ?

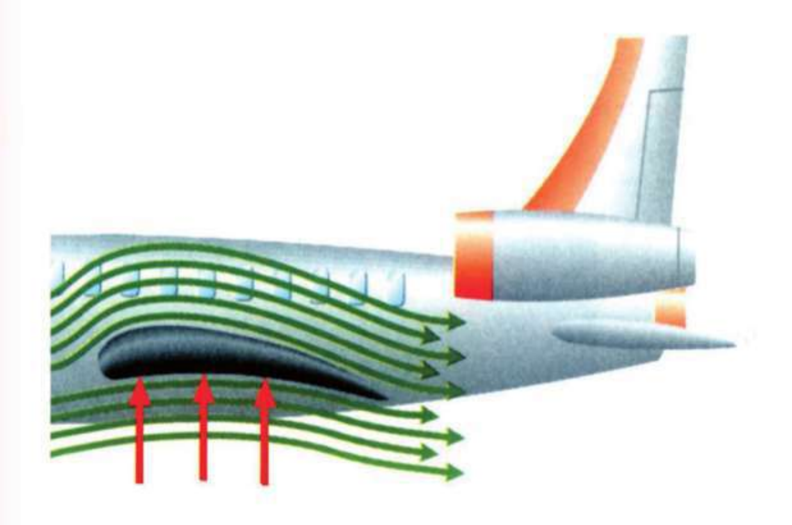
The motion of a plane forms a region over the wings where the pressure is less than that below the wings. An upthrust develops to balance out with the weight of the plane (Fig 4-6). This phenomenon is called Bernoulli's Effect.



a) A plane flies and does not fall




b) A region of rarefaction forms above the plane's wing



c) The force due to the difference in pressure pushes the plane up

Fig (4-6)

How a plane flies

**Solution**

The aorta cross section is given by

$$A_1 = \pi r_1^2 = \pi (0.007)^2 \text{ m}^2$$

The collective cross section for 30 main arteries is given by

$$A_2 = \pi r_2^2 \times 30$$

$$= \pi (0.0035)^2 \times 30 \text{ m}^2$$

$$A_1 v_1 = A_2 v_2$$

$$\pi (0.007)^2 (0.33) = \pi (0.0035)^2 (30) v_2$$

$$v_2 = \frac{4 \times 0.33}{30} = 0.044 \text{ m/s}$$

Thus, the velocity of the blood in the main arteries is 0.044 m/s. Consequently, the blood velocity in capillaries is much smaller, which gives time for the tissues to exchange oxygen and carbon dioxide as well as nutrients and excretion products. Divine wonder is countless.

## Viscosity

We observe viscosity as follows :

1) We hang two funnels each on a stand and put a beaker under each. We pour alcohol in one funnel and a similar volume of glycerine in the other, and observe the velocity of flow of each. We notice that the flow velocity of alcohol is higher than that of glycerine.

2) Take two similar beakers, one containing a certain volume of water and the other an equal volume of honey .Stir the liquid in both beakers with a glass rod .Which of the two liquids is easier to stir ? Then, we remove the rod .We notice that :

a) Water is easier to stir, which means that water resistance to the glass rod is less than the resistance of honey.

b) The motion in honey stops almost immediately after we remove the rod, while it continues for a little while longer in water .

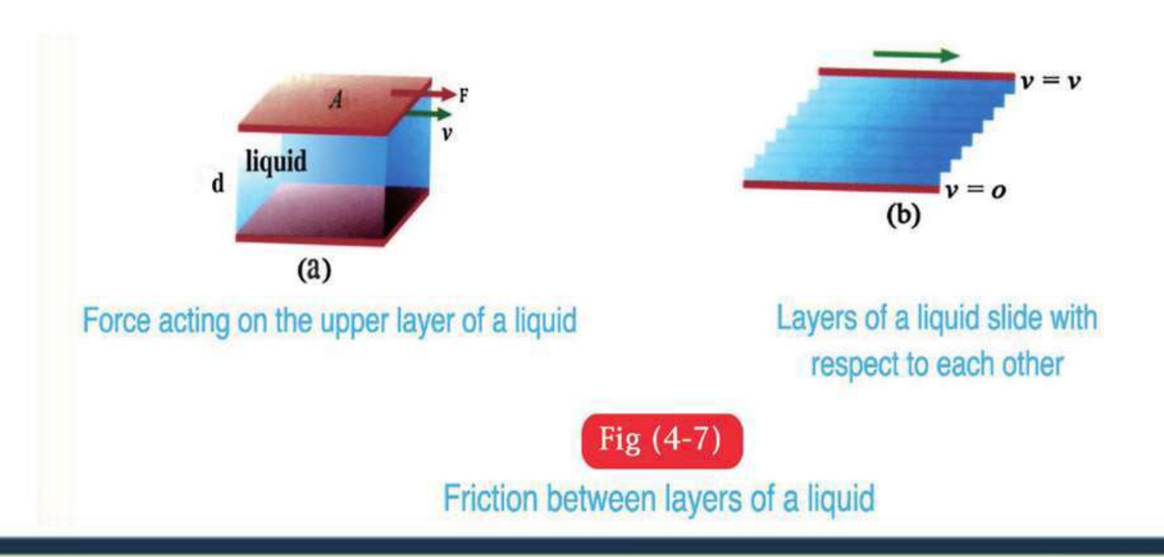
3) We take two long similar measuring cylinders and fill them to the end, one with water and the other with glycerine. Then , take two similar steal balls and drop one in each liquid, and record the time each ball takes in each liquid to hit the bottom. We observe that the time in water is less. Thus, the glycerine resistance to the ball motion is greater .

We, thus, conclude :-

1) Some liquids such as water and alcohol offer little resistance to the motion of a body in them, and are easy to flow. We say they have low viscosity

2) Other liquids such as honey and glycerine are not as easy to move through ,i.e.,they offer high resistance to body motion, and are said to have high viscosity.

To interpret viscosity, imagine layers of liquid trapped between two parallel plates, one stationary and the other moving with velocity v **(Fig 4-7).** The liquid layer next to the stationary plate is stationary, while the layer next to the moving plate is moving at v. The layers in between move at velocities varying from o to v. The reason for this is as follow :




Force acting on the upper layer of a liquid

Layers of a liquid slide with respect to each other

Fig (4-7)

Friction between layers of a liquid

a) Friction forces exist between the lower plate and the liquid layer in contact with it. This force is due to the adhesive forces between the molecules of the solid surface and the contacting liquid molecules. This leads to zero velocity of the layer in contact with the stationary plate. Similarly, the upper layer moves at the same velocity of the upper plate.

b) The existence of another friction (shear) force between each liquid layer and the adjacent one, which resists the sliding of the liquid layers with respect to each other. This produces a relative change in velocity between any two adjacent layers. Thus, viscosity is the property responsible for resisting the relative motion of liquid layers .This type of flow is called nonturblent viscous laminar flow (or viscous steady flow), since no vortices occur.

## Coefficient of Viscosity

Referring to Fig (4 – 6), we find that for the moving plate to maintains its constant velocity, a force F must exist. This force is directly proportional to both velocity and area of the moving plate A, and inversely proportional to the distance between the plates d.

$$F \propto \frac{Av}{d}$$

$$F = \eta_{vs} \frac{Av}{d} \qquad (4 - 3)$$

where $\eta_{vs}$ (Eta) is a constant of proportionality called viscosity coefficient, given by

$$\eta_{vs} = \frac{Fd}{Av} = \frac{F}{Av/d} \qquad (4 - 4)$$

The coefficient of viscosity ( $Ns/m^2$ or kg/m s ) may be defined as :

**the tangential force acting on unit area, resulting in unit velocity difference between two layers, separated by unit distance apart .**

## Applications of Viscosity

### 1-Lubrication :

Metallic parts in machines have to be lubricated from time to time. This process leads to:

a) reduction of heat generated by friction.

b) protecting machine parts from corrosion (wear).

Lubrication is carried out using highly viscous liquids. If we use water (low viscosity), it will soon seep away or sputter from the machine parts due its low adhesive forces. Therefore, we must use liquids with high adhesive (high viscosity), so they remain in contact with the moving machine parts.

### 2-Moving vehicles

When a car attains its maximum speed, the total work done by the machine which is supplied by the burnt fuel, acts most of the time against air resistance and the forces of friction between the tires and the road. At relatively low and medium velocities, air resistance to moving bodies resulting from air viscosity is directly proportional to the velocity of the moving body. When the velocity exceeds a certain limit, then the air resistance is proportional to the square velocity rather than the velocity,leading to a noticeable increase in fuel consumption. Therefore, it is advisable not to exceed such a limit ( 80 – 90 km/h )

### 3-In medicine :

**Blood precipitation rate :** when a ball undergoes a free fall in a liquid, it is under three forces : its weight, buoyancy of the liquid and friction between the ball and the liquid due to viscosity. It is found that such a ball attains a final velocity which increase with its radius.

This is applied in medicine by taking a blood sample and measuring its precipitation rate. The doctor may then decide if the size of red blood cells is normal or not. In the case of rheumatic fever and gout, red blood cells adhere together, and therefore, their volume and radius increase and the sedimentation ( precipitation ) rate increases. In the case of anemia, the precipitation rate is below normal, since the red cells break up. Hence, their volume and radius decrease .

## In a Nustshell

### Definitions and Basic Concepts:

- A fluid is a substance that can flow and does not take a definite form.
- A steady flow in a tube requires :

  a) the liquid fills the tube completely.

  b) the quantity of liquid entering the tube at one end is equal to the quantity of liquid emerging at the other end in the same time.

- Viscosity is the property where resistance (or friction) exists between the layers of the liquid hindering the easy sliding of these layers.
- Coefficient of viscosity ($kgm^{-1}s^{-1}$) is the tangential force acting on unit area resulting in unit difference of velocity between two layers separted by unit distance apart.

### Basic laws :

- The volume rate of a liquid flowing with velocity v across area A in unit time is $Q_v = Av$
- The mass rate of flow is $Q_m = \rho Q_v$
- The continuity equation is $A_1 v_1 = A_2 v_2$
- The coefficient of viscosity for a fluid is given by:

$$\eta_{vs} = \frac{Fd}{Av}$$

where F is the tangential force between two layers of a liquid, A is the area of the moving layer, v is the velocity of the moving layer and d is the distance between the moving and stationary layers.

## Questions and Drills

### I) Define

1) fluid 2) viscosity 3) coefficient of viscosity

### II) Essay questions

1) Prove that the velocity of a liquid at any point in a tube is inversely proportional to the cross sectional area of the tube at that point.

2) Explain the property of viscosity.

3) Illustrate some applications of viscosity.

### III) Drills

1- Water flows in a horizontal hose at a rate of 0.002 $m^3/s$, calculate the velocity of the water in a pipe of cross sectional area $1 cm^2$. (20 m/s)

2- Water flows in a rubber hose of diameter 1.2 cm with velocity 3 m/s. Calculate the diameter of the hose if the velocity of the emerging water is 27 m/s. (0.4cm)

3- A main artery of radius 0.035 cm branches out to 80 capillaries of radius 0.1 mm. If the velocity of blood through the artery is 0.044 m/s ,what is the velocity of blood in each of the capillaries? (0.0067 m/s)

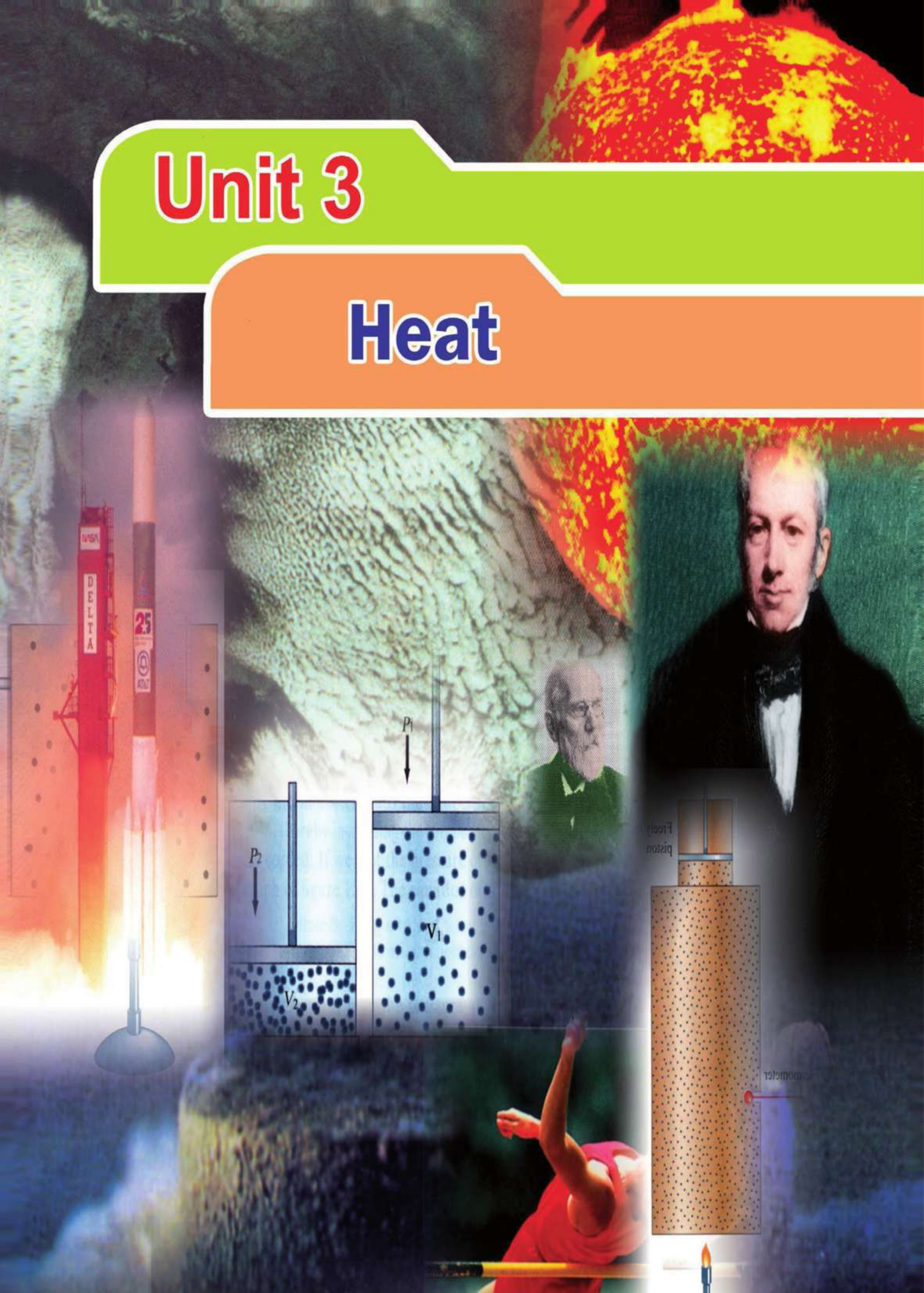
4- The cross sectional area of a tube at point A is 10 $cm^2$ and at point B is 2$cm^2$. If the velocity of water at A is 12 m/s, what is the velocity at B ? (60 m/s)

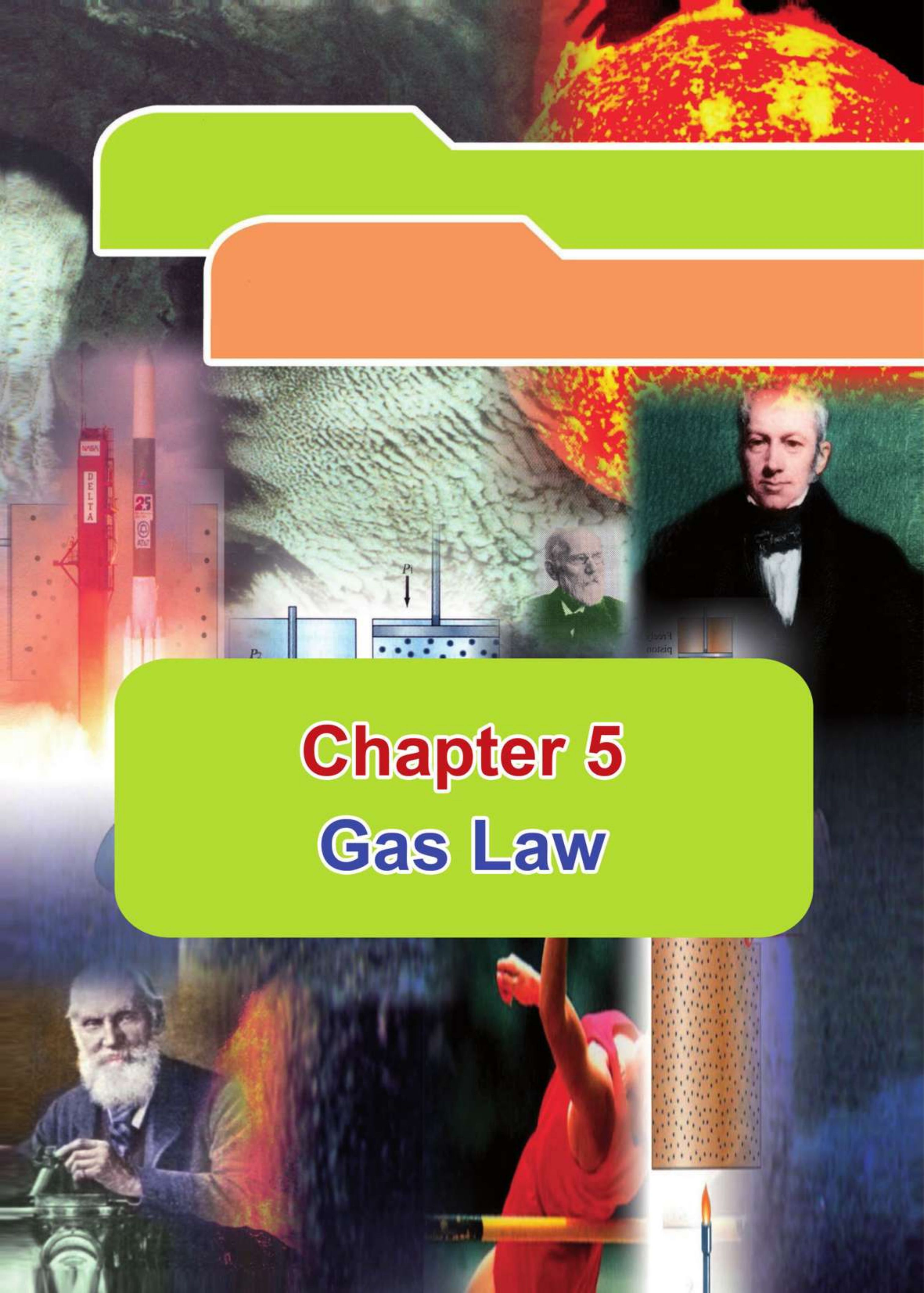
5- The cross sectional area of a water pipe at the ground floor is $4 \times 10^{-4} m^2$. The velocity of the water is 2 m/s. When the pipe tapers to a cross sectional area of $2 \times 10^{-4} m^2$ at the end, calculate the velocity of the flow of water at the upper floor.

(4m/s)

# Unit 3

## Heat




Chapter 5
Gas Law

# Chapter (5) The Gas Laws

## Overview

It can be shown that gas molecules are in continuous random motion called Brownian motion as follows:

If we examine candle smoke through the microscope,we notice that the smoke particles move randomly. The motion of the carbon particles is called Brownian motion, after Brown, an English botanist who discovered for the first time in 1827 that tiny pollen grains suspended in water moved randomly.

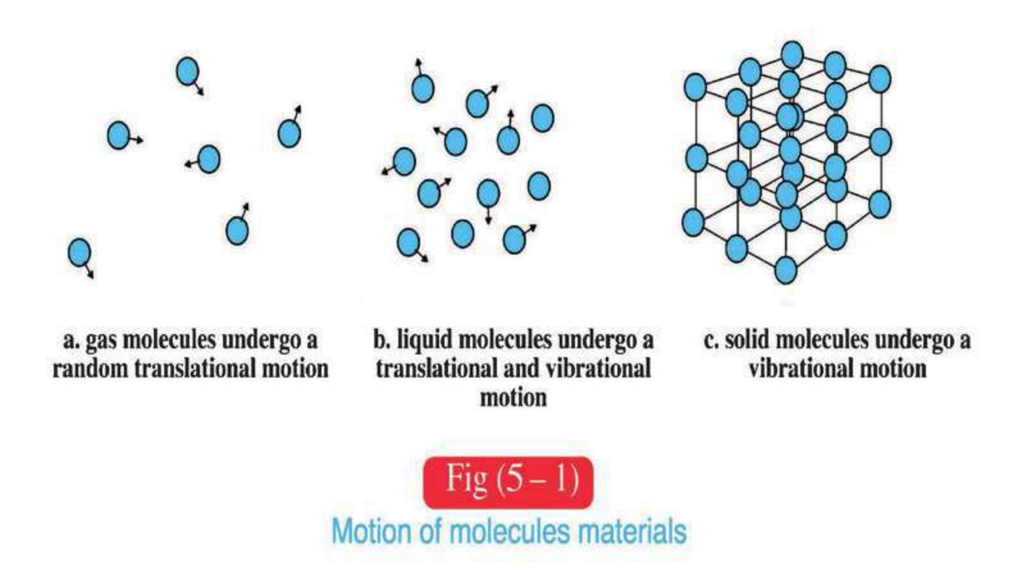



Fig (5 – 1)
Motion of molecules materials

## Interpretation of Brownian motion

Air (gas) molecules move in a haphazard (random) motion in all directions with different velocities. During their motion, they collide with each other and collide with smoke particles. Due to the resultant force on a smoke particle, it will move in a certain direction through a short distance and so on, always moving, colliding, and changing direction. The reason for this is that the gas molecules are in a free motion (due to heat) and in constant collision, so they change their direction randomly (Fig 5-1).

It can be concluded that :

- Gas molecules are in a state of continuous random motion.
- In their motion, they collide with each other and collide with the walls of the container
- The distance between the molecules is called intermolecular distance (more or less constant for different gases at the same conditions).

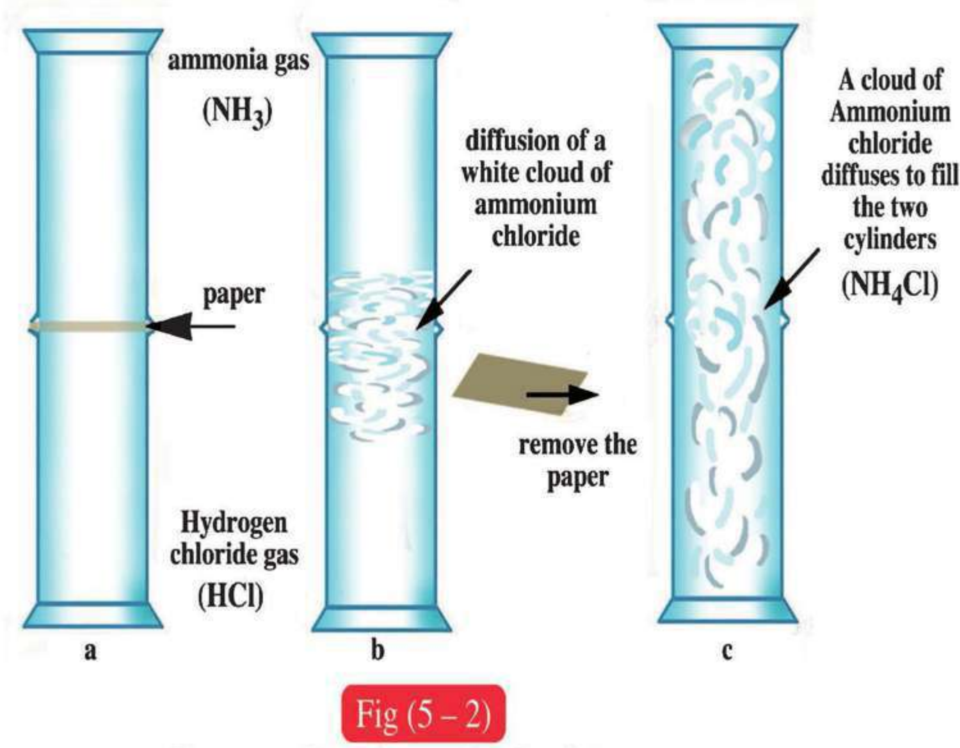



Fig (5 – 2)

Presence of gas intermoleculor distances

The evidence of the existence of intermolecular distances can be shown as follows: When a graduated cylinder filled with ammonia gas is placed upside down on another cylinder filled with hydrogen chloride gas (Fig 5-2), a white cloud of ammonium chloride is formed, then it grows and diffuses until it occupies all the space within the two cylinders.

This can be explained as follows. Hydrogen chloride gas molecules - in spite of their higher density - diffuse upwards, through spaces separating ammonia gas molecules, where they combine together forming ammonium chloride molecules, which diffuse to fill the

upper cylinder. Also, ammonia gas molecules - in spite of their lower density - diffuse downwards through spaces separating hydrogen chloride gas molecules, where they combine forming ammonium chloride molecules, which diffuse to fill the lower cylinder.

Accordingly, we can conclude that there are large spacings separating the gas molecules, known as intermolecular spacings. This is to be tied to the compressibility of gases. These large intermolecular spacings allow gas molecules to get packed together when pressed. Thus, a volume occupied by a gas decreases with increased pressure.

## Gas Laws

Experiments performed to evaluate the thermal expansion of a gas are complicated. The volume of a gas is affected by changes in pressure as well as by temperature. This difficulty does not arise in the case of solids or liquids, as these are very much less compressible.

In order to make a full study of the behavior of a gas, as regards volume, temperature and pressure, three separate experiments have to be carried out to investigate the effect of each pair, respectively, i.e., we study the relation between two variables only, keeping the third constant.These experiments are

1- The relation between the volume and pressure at constant temperature (Boyle's law).

2- The relation between the volume and temperature at constant pressure (Charle's law).

3- The relation between the pressure and temperature at constant volume (Pessure law or Jolly's law).

We are going to study each of these three relations.

## Firstly: the relation between the volume and pressure of a gas at constant temperature (Boyle's law) :

To study the relation between the volume of a fixed mass of gas and its pressure at constant temperature, the apparatus shown in Fig (5- 3) is used. It consists of a burette (A) connected by a length of rubber tube to a glass reservoir (B) containing a suitable amount of mercury. (A) and (B) are mounted side by side onto a vertical stand attached to a base provided by three screws with which the stand is adjusted vertically. The reservoir (B) is movable along the stand either upwards or downwards and can be fixed at any desired position.

### Procedure:

1- The tap (A) is opened and the reservoir (B) is raised until the mercury level in burette A is about half full, taking into account that the mercury levels are the same in both sides. (Fig 5-3a) .

2- The tap (A) is then closed. The volume of the enclosed air is measured, let it be $(V_{ol})_1$. Its pressure is also measured, let it be $P_1$, which equals the atmospheric pressure $P_a$ (cmHg) which may be determined using a barometer.

3- The reservoir (B) is then raised a few centimetres and the volume of the enclosed air is measured $(V_{ol})_2$. The difference

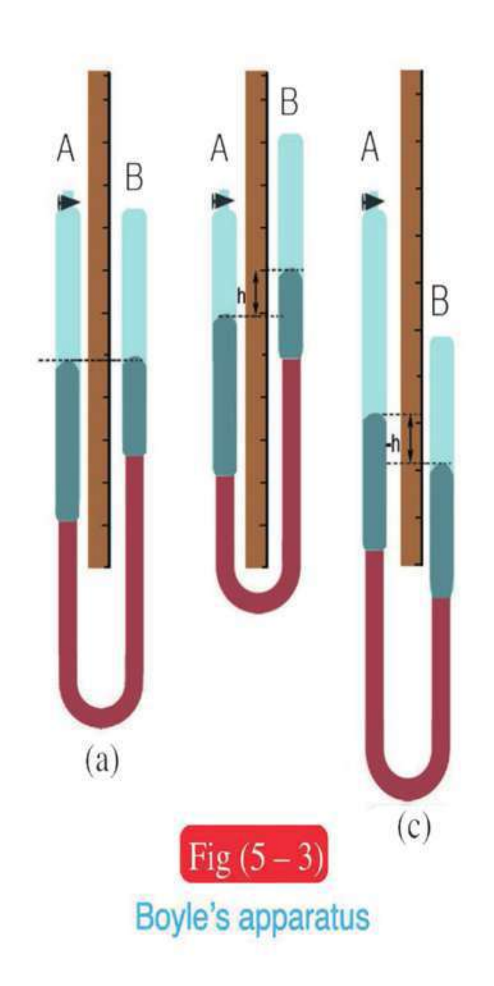



Fig (5 – 3)
Boyle's apparatus




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between the two levels of mercury in both sides (h) is determined . In this case, the pressure of the enclosed air (cmHg) is $P_2 = P_a + h$ (Fig 5 - 3 b).

4- Repeat the previous step by raising the reservoir (B) another suitable distance and measure $(V_{ol})_3$ and $P_3$ in the same manner.

5- The reservoir (B) is then lowered until the mercury level in (B) becomes lower than its level in (A) by a few centimeters. Then, the volume of the enclosed air is measured $(V_{ol})_4$ and its pressure $(P_4)$ is determined $P_4 = P_a - h$, where h is the difference between the two levels of mercury in both sides (Fig 5-3c).

6- The previous step is repeated once more by lowering (B) another suitable distance. Then $(V_{ol})_5$ and $P_5$ are measured in the same manner.

7- Plot the volume of the enclosed air $(V_{ol})$ and the reciprocal pressure ( $\frac{1}{P}$ ).We obtain a straight line (Fig 5 - 4) Thus, we can conclude that:

$$V_{ol} \propto \frac{1}{P}$$

,i.e.,the volume of a fixed mass of gas is inversely proportional to the pressure, provided that the temperature remains constant. This is "Boyle's law".

Boyle's law can be written in another form, as:

$$V_{ol} = \frac{const}{P}$$

$$\boxed{PV_{ol} = Const.} \qquad (5 - 1)$$

i.e., is : **at a constant temperature, the product $PV_{ol}$ of any given mass of a gas is constant.**

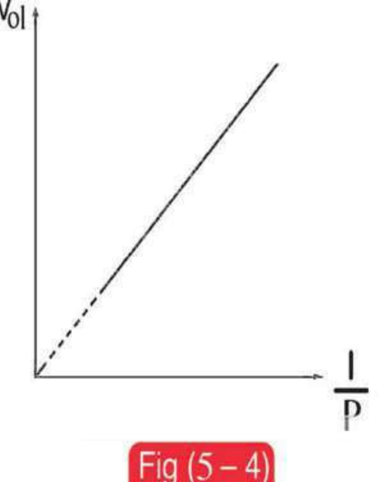



Fig (5 – 4)
Relation between volume and reciprocal pressure of gas

## The effect of temperature on the volume of a gas at constant pressure:

We have already known that gases contract by cooling and expand by heating. But, does the same volume of different gases at constant pressure expand by the same amount?

To show this, let us do the following experiment:

1- Take two flasks of exactly equal volume, each fitted with a cork through which a tube bent 90° is inserted. In each tube, there is a thread of mercury of length 2 or 3 cm. Fill one of the flasks with oxygen and the other with carbon dioxide or air. Submerge the two flasks in a vessel filled with water as shown in Fig (5 - 5).

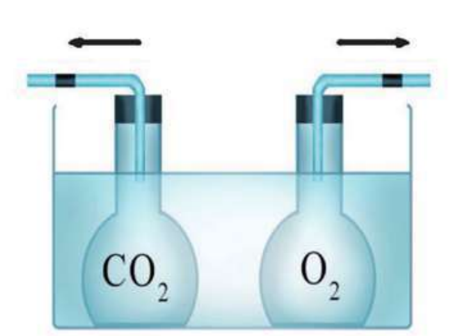



Fig (5 – 5)
Effect of temperature on the volume of a gas at constant pressure

2- Pour hot water into the vessel and notice the distance moved by the mercury thread in both tubes. You will find that these distances are equal. This indicates that equal volumes of different gases expand equally when heated through the same temperature rise . In other words, they have the same volume expansion coefficient.

**Volume expansion coefficient of a gas at constant pressure $\alpha_v$ is defined as :**

**"It is the increase in volume at constant pressure per unit volume at 0°C for 1°C rise in temperature ".**

## Secondly: the relation between the gas volume and its temperature at constant pressure (Charle's law) :

To investigate the relation between the gas volume and its temperature at constant pressure, the apparatus shown in Fig(5 - 6a) is used. It consists of a capillary glass tube 30 cm long and about 1 mm diameter with one end closed. The tube contains a short pellet of mercury enclosing an amount of air inside it whose length is measured by a ruler stand. The apparatus is equipped with a thermometer inside a glass envelope. We follow the folowing procedure:

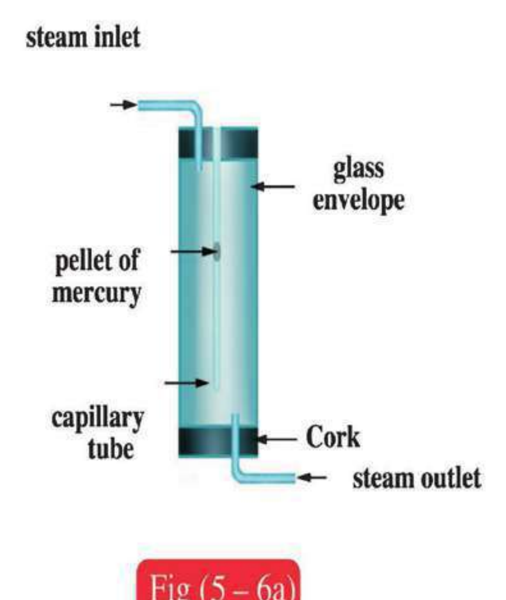
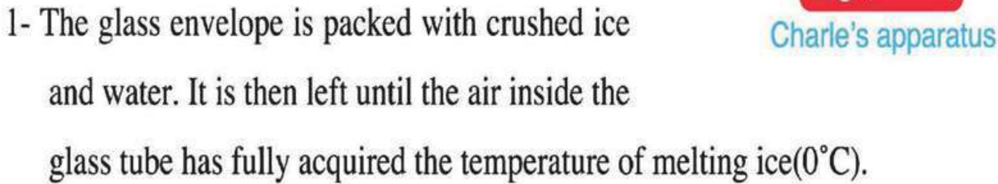



Fig (5 – 6a)
Charle's apparatus



1- The glass envelope is packed with crushed ice and water. It is then left until the air inside the glass tube has fully acquired the temperature of melting ice(0°C).

2- The length of the enclosed air is then measured, and since the tube has a uniform cross-section, the length of the encloscd air is taken as being proportional to its volume $(V_{ol})_{0^\circ c}$

3- The ice and water are removed from the envelope and steam is passed through the top and out at the bottom for several minutes to be sure that the temperature of air becomes 100°C . Then, the length of the enclosed air is measured. It is taken as a measure of its volume $(V_{ol})_{100^\circ C}$.

4- A relation between $V_{ol}$ and t°C is plotted (Fig 5-6b). We see that such a relation is a straight line,which if extended will intersect the abscissa at -273°C .

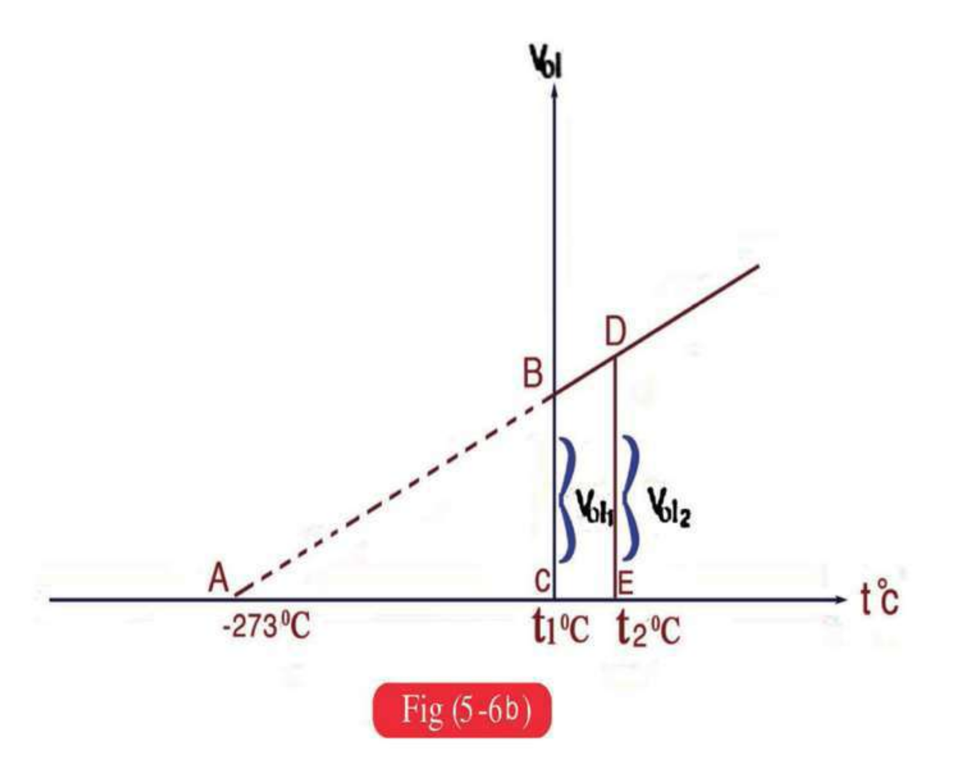



Fig (5-6b)

5- The volume coefficient of air at constant pressure $\alpha_v$ is then determined using the relation.

$$\alpha_V = \frac{(V_{ol})_{100°C} - (V_{ol})_{0°C}}{(V_{ol})_{0°C} \times (100°C)} \quad (5\text{-}2)$$

The value of $\alpha_v$ obtained experimentally is 1/273 per unit degree rise in temperature degree Kelvin.

Since equal volumes of different gases expand equally at constant pressure, the volume coefficient of expansion of all gases have the same value .

This result was formulated by Charle as follows:

## Charle's law :

**"The volume of a given mass of gas,kept at constant pressure,expands by $\frac{1}{273}$ of its value at 0°C per each degree rise in temperature.This value is the same for all gases".**

**Note: 1 degree rise in Kelvin = 1 degree rise in Celsius, (why?).**

## The effect of temperature on the pressure of a gas at constant volume:

1- To investigate how the pressure of a gas depends on temperature, the apparatus shown (Fig 5-7a) may be used. The gas under test is confined in a flask by mercury in a U tube. The flask is fitted with a cork. The surfaces of mercury in the two branches (A) and (B) have the same level at x,y. Thus, the pressure of the enclosed air is atmospheric. We then determine the temperature of air. Let it be $t_1$°C.

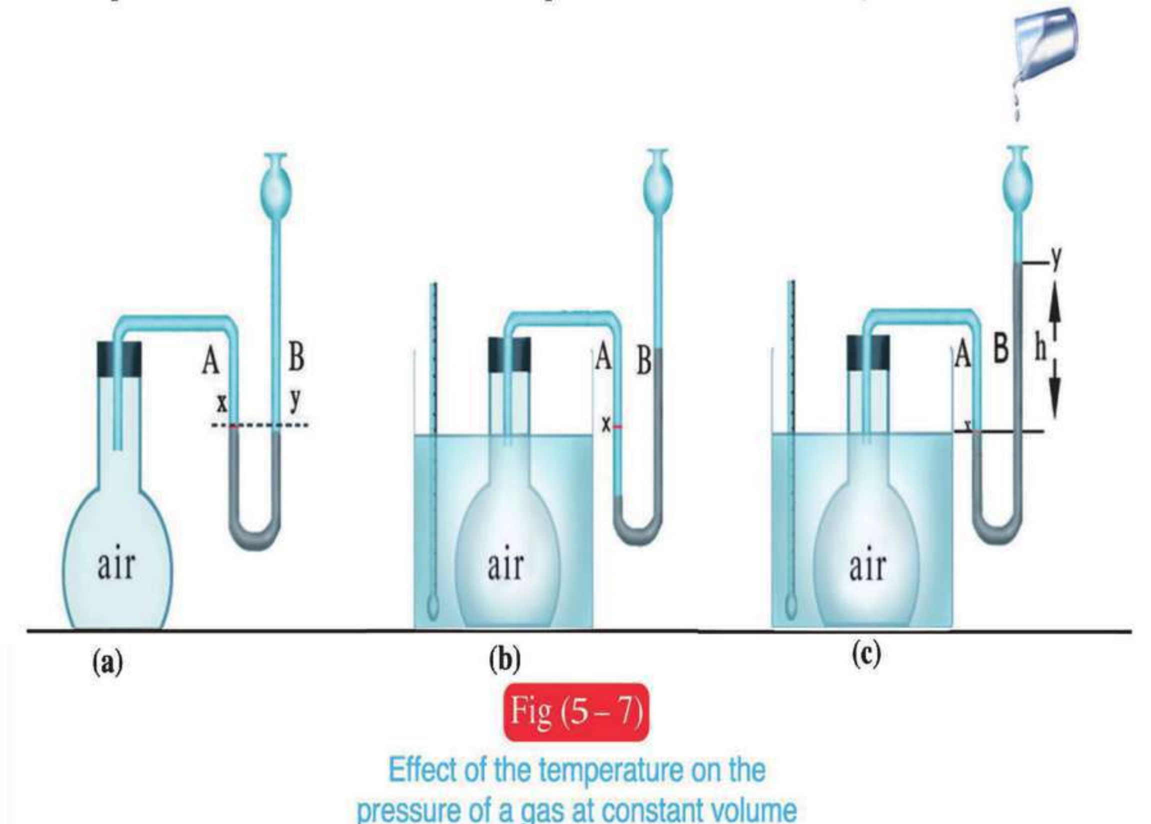



Fig (5 – 7)

Effect of the temperature on the pressure of a gas at constant volume

2- Submerge the flask in a vessel containing lukewarm water at $t_2$°C. You will notice that the level of mercury decreases in branch A, while it rises in branch B (Fig 5-7b).

3- We pour mercury in the funnel C, until the level of mercury in branch A returns to the mark x then the volume of the enclosed air in the flask at $t_2$°C is equal to the volume at $t_1$°C (Fig 5-7c).

4- We notice that the surface of mercury in branch B exceeds that in branch A by an amount" h" (cm). This means that the pressure of the enclosed air has increased as a result of the temperature rise from $t_1$°C to $t_2$°C by an amount equal h (cmHg)( Fig 5-7c).

5- Repeating this experiment several times for different gases and measuring the amount of increase of gas pressure at constant volume for the same rise in temperature, we find:

a- At constant volume, the pressure of a given mass of gas increases by increasing temperature.

b- At constant volume, equal pressures of gases increase equally, when heated through the same range of temperatures. We define the pressure expansion coefficient of a gas at constant volume ($\beta_P$) as :

**"It is the increase in gas pressure at constant volume per unit pressure at 0°C for cm degree rise in temperature". It is found to be the same for all gases.**

## Thirdly: the relation between the pressure and temperature of a gas at constant volume (pressure law ):

It was found experimentally that the increase in gas pressure is directly proportional to the initial pressure at 0°C ($P_{0°C}$) as well as to the rise in its temperature, ($\Delta t°C$). This is expressed as follows:

$$\Delta P \propto P_{0°C}\,\Delta (t°C)$$

$$\Delta P = \beta_P P_{0°C}\,\Delta (t°C)$$

$$\beta_P = \frac{\Delta P}{P_{0°C}\,\Delta (t°C)} \qquad (5\text{-}3)$$

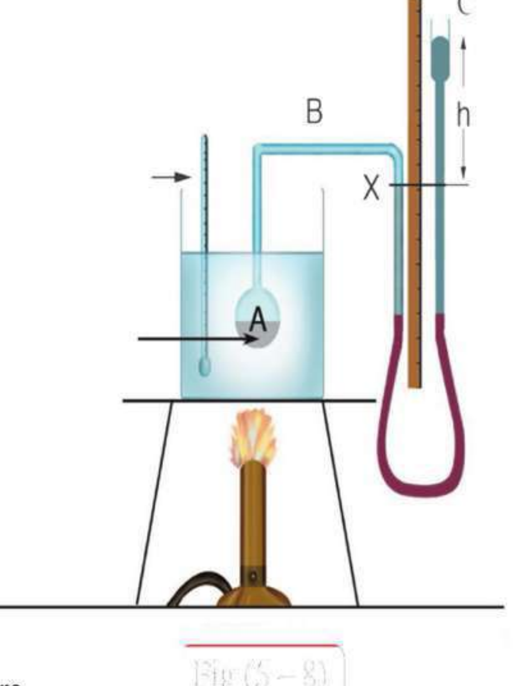


Fig (5 – 8)

where $\beta_P$ is a constant value. It is the pressure

expansion cofficient of a gas with temperature, at constant volume.It is the same for all gases.

To measure $\beta_p$ of a gas at constant volume, Jolly's apparatus shown in Fig (5-8) is used. It consists of a glass bulb (A). The bulb is joined to a capillary tube (B) bent in the form of two right angles. The bulb and the tube are mounted on a vertical ruler attached to a board which is fixed on a horizontal base provided with 3 leveling screws.

The capillary tube (B) is connected to a mercury reservoir (C) by means of a rubber tube.

We follow the following procedures:

1- Determine the atmospheric pressure ($P_a$) using a barometer.

2- Pour mercury in (A) to 1/7 of its volume to compensate for the increase in its volume when heated, so that the volume of the remaining part is still constant, (the volume expansion coefficient of mercury is seven times the volume expansion coefficient of glass).

3- Submerge reservoir (A) in a beaker filled with water and pour mercury in the free end (C), until it rises in the other branch to mark (X).

4- Heat water in the vessel to the boiling point and wait until the temperature settles, and the mercury level in the branch connected to the reservoir stops decreasing.

5- Move the free end (C) upwards until the the mercury level in the other branch rises to the same mark X. Then, measure the difference in height between the mercury levels in the two branches (h). From this, determine the pressure of the enclosed air P, which is equal to the atmospheric pressure (cm Hg) plus h, i.e., $P=P_a+h$

6- Move the branch (C) downwards and stop heating. Then let the reservoir cool down to nearly 90˚C. Then move the branch (C) upwards until the mercury level in the branch connected to the reservoir rises to mark X.

Then determine the temperature and the difference in height between the mercury levels in both branches. From this we calculate the pressure of the enclosed air in this case.

7- Repeat at different temperatures and determine the pressure of the enclosed gas in each case.

8- Plot the relation between pressure (vertical axis) and temperture (horizontal axis). We find that the relation is a straight line.

We calculate the pressure coefficient of a gas at constant volume from the equation:

$$\beta = \frac{P_{100^\circ C} - P_{0^\circ C}}{P_{0^\circ C} \times 100^\circ C} \quad (5\text{-}4)$$

The value of $\beta_P$ obtained experimentally is $\frac{1}{273}$ per unit degree rise in temperature. Moreover, the same result is obtained for all gases.

From the results above, we conclude that:

**At consant volume, the pressure of a given mass of any gas changes by $\frac{1}{273}$ of its value at 0°C for every degree change of temperature.**

## The absolute zero (zero kelvin)

Using the apparatus shown in Fig(5-7) to measure the volume of the enclosed air at different temperatures, the result may be plotted as a graph relating temperature (t°C) (horizontal axis) and volume (vertical axis). We find it to be a straight line. When extended to intersect the the horizontal axis, we find that the point of intersection is at - 273°C (Fig 5 - 9).

We can plot the results obtained using Jolly's apparatus. The graph obtained shows a relation between the temperature (t°C) and pressure. It will be as shown in Fig.(5-10). Extrapolating backwards the straight line, it cuts the temperature axis at (-273°C). This suggest that -273°C is the lowest temperature attainable theoretically or the absolute zero, i.e. ,zero Kelvin. We may, thus, define the absolute zero as:

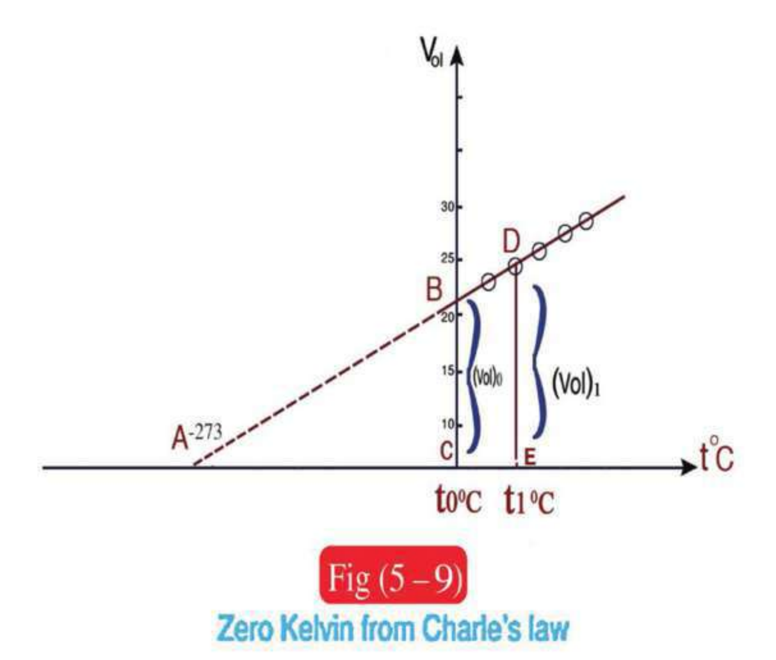



Fig (5 – 9)

Zero Kelvin from Charle's law

It is the temperature at which the volume and the pressure of an ideal gas disappear. The mperature on the Kelvin scale has a positive value only, while the Celsius scale inges between positive and negative values.

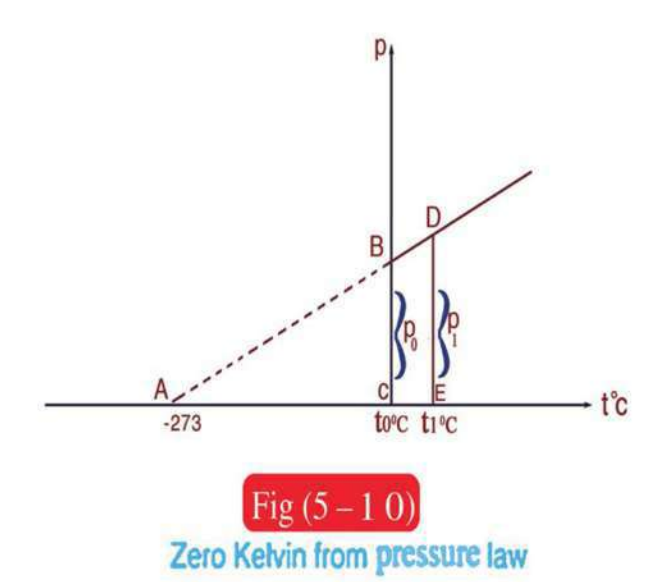



Fig (5 – 1 0)

Zero Kelvin from pressure law

Learn at Leisure

## The absolute zero
## (The Kelvin scale)

We can replot Figs.(5-9 , 5-10) such that the abcissa is absolute temperature. We obtain Figs.(5-11 , 5-12). At absolute zero, $V_{ol} = 0$ and $P = 0$. In fact, with extreme cooling, the material is no longer gaseous, but transforms to liquid or often solid. Hence, it does not obey the gas laws. We may define the ideal gas as that gas whose volume and pressure vanish at absolute zero. It is to be noted that we have neglected in deriving the gas laws the forces of attraction among the molecules and the size of the molecules with respect to the size of the container. This is called the perfect gas condition.

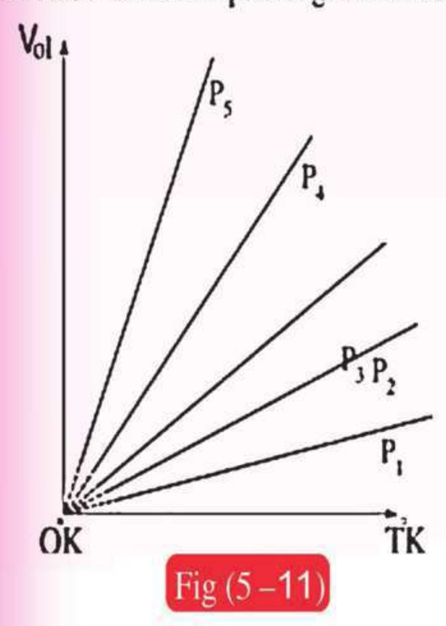



Fig (5 –11)

Relation between volume and temperature at constant pressure

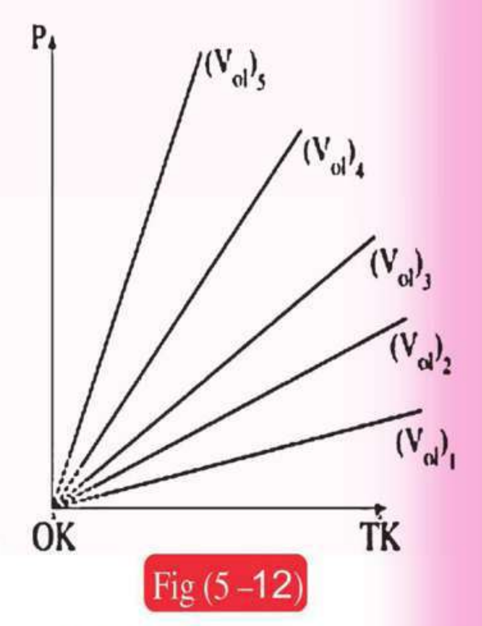



Fig (5 –12)

Relation between pressure and temperature at constant volume

To find the relation between the Celsius and Kelvin scales, we note.

O°K corresponds to (- 273°C)

O°C corresponds to ( 273°K)

100°C corresponds to ( 373°K)

In general,

$$T(°K) = 273 + t(°C) \quad (5\text{-}5)$$

## Other Forms of Charle's and Jolly's (pressure) laws :

1- Refering to (Fig 5-9),

we note that the triangles ABC and ADE are similar. Therefore:

$BC = (Vol)_1$

$DE = (Vol)_2$

$AC = T_1$

$AE = T_2$

$V_{ol} \propto T$

$$\frac{V_{ol}}{T} = \text{const.}$$

$$\therefore \frac{(V_{ol})_1}{T_1} = \frac{(V_{ol})_2}{T_2} \qquad (5-6)$$

Thus, at constant pressure, the volume of a fixed mass of gas is directly proportional to its temperature on the Kelvin scale. This is another formulation of Charle's law.

2- Using Fig.(5-10), the following relation can be obtained in a similar way:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad (5-7)$$

That is

$$\frac{P}{T} = \text{const}$$

or $P \propto T$

Thus, at constant volume, the pressure of a fixed mass of gas is directly proportional to its temperature on the Kelvin scale. This is another form of pressure (Jolly's) Iaw.

## General gas law:

We have seen that the gas behavior can be described by three variables, namely, volume, pressure and temperature. The equation relating these three variables together is the general gas law.

From Boyle's law : $V_{ol} \propto \frac{1}{P}$

From Charle' law : $V_{ol} \propto T$

Therefore : $V_{ol} \propto \frac{T}{P}$

From which $V_{ol} = const \times \frac{T}{P}$

Therefore : $\therefore \frac{P V_{ol}}{T} = const$

$$\frac{P_1 (V_{ol})_1}{T_1} = \frac{P_2 (V_{ol})_2}{T_2} \quad (5\text{-}8)$$

This is the general gas law, which satisfies as well Jolly's law.

### Go Further

For more knowledge about this topic you can refer to the Egyptian Knowledge Bank (EKB) through the opposite links



**Examples :**

1- The volume of a gas at 0°C is 450 cm$^3$. Find its volume at 91°C, assuming that pressure is constant.

**Solution:**

$$\frac{(V_{ol})_1}{(V_{ol})_2} = \frac{T_1}{T_2}$$

$$\frac{450}{(V_{ol})_2} = \frac{273}{273 + 91}$$

$$(V_{ol})_2 = \frac{450 \times 364}{273} = 600 \text{ cm}^3$$

2- Half a liter of hydrogen is heated from 10°C to 293°C. Find its volume assuming that the pressure is constant.

**Solution:**

$$\frac{(V_{ol})_1}{(V_{ol})_2} = \frac{T_1}{T_2}$$

$$\frac{500}{(V_{ol})_2} = \frac{273 + 10}{273 + 293}$$

$$(V_{ol})_2 = \frac{500 \times 566}{283} = 1000 \text{ cm}^3 = 1 \text{ liter}$$

3- The pressure of a gas at 26°C is 59.8 cmHg. Find its pressure at 130°C, assuming that the volume is constant.

**Solution:**

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\frac{59.8}{P_2} = \frac{273 + 26}{273 + 130}$$

$$P_2 = \frac{59.8 \times 403}{299} = 80.6 \text{ cmHg}$$

4- The volume of a gas at 27°C under a pressure of 60 cm Hg is 380 cm³. Find its volume at normal temperature and pressure (S.T.P).

**Solution:**

Normal (or standard) pressure and temperature means at 0°C(or 273°K) and under a pressure of 76 cm Hg.

$$\frac{P_1(V_{ol})_1}{T_1} = \frac{P_2(V_{ol})_2}{T_2}$$

$$\frac{60 \text{ X } 380}{300} = \frac{76 \times (V_{ol})_2}{273}$$

$$(V_{ol})_2 = \frac{60 \text{ X } 380 \text{ X } 273}{76 \text{ X } 300} = 273 \text{ cm}^3$$

5- The volume of a vessel containing nitrogen gas is 15 liters under a pressure of 12 cm Hg. The volume of another vessel containing oxygen gas is 10 liters under a pressure of 50 cm Hg. The two gases are mixed in another vessel of volume 5 liters. Find the pressure of the mixture if the temperature of the two gases and the mixture is kept constant.

**Solution:**

Every gas after mixing occupies a volume of 5 liters. To find the pressure of the nitrogen gas we use the equation:

$$P V_{ol} = P_1 (V_{ol})_1$$

$$\therefore 12 \times 15 = P_1 \times 5$$

$$P_1 = 36 \text{ cmHg}$$

To find the pressure of oxygen gas we use the equation:

$$PV_{ol} = P_2 (V_{ol})_2$$

$$P_2 = \frac{10 \times 50}{5} = 100 \text{ cm Hg}$$

Since the pressure of the mixture equals the sum of the pressures of each gas, hence,

$$P = P_1 + P_2 = 36 + 100 = 136 \text{ cm Hg}$$

## In a Nutshell

**1- Definitions and Basic Concepts :**

- Gas molecules are in continuous random motion and collide with each other and with the walls of the container.
- There are spaces between gas molecules called intermolecular distances.
- Boyle's law: At constant temperature, the volume of a fixed mass of a gas is inversely proportional to its pressure.
- Charles's law: At constant pressure, the volume of a given mass of a gas expands by $\frac{1}{273}$ of its volume at 0°C per each degree rise in temperature. Alternatively, at constant pressure, the volume of a given mass of gas is directly proportional to its temperature on the Kelvin scale.
- Pressure law : At constant volume the pressure of a given mass of gas rises by $\frac{1}{273}$ of its initial pressure at 0°C per each degree rise in temperature. Alternatively, at constant volume, the pressure of a given mass of gas is directly proportional to its temperature on the Ke1vin scale.
- Pessure coefficient of all gases at constant volume = volume coefficient of expansion at constant pressure for all gases = $\frac{1}{273}$ = 0.003663.
- Absolute temperature (Kelvin scale) = 273 + t°C.

**2- Basic laws:**

If $V_{ol}$ is the volume of a given mass of a gas, P its pressure and T its temperature on the Kelvin scale:

- Boyle's law: $PV_{ol}$ = const (at constant temperature)

- Charle's law $\frac{V_{ol}}{T} = const$ (at constant pressure).
- Pressure law : $\frac{P}{T} = const.$ (at constant volume).
- (General law of gases:) $\frac{P_1(V_{ol})_1}{T_1} = \frac{P_2(V_{ol})_2}{T_2}$
- It means that $\frac{PV_{ol}}{T} = const$
- Volume expansion coeffieint of a gas at constant pressure per unit degree rise in temperature $\alpha_v$ is given by:

$$\alpha_v = \frac{(V_{ol})_{t^\circ C} - (V_{ol})_{0^\circ C}}{(V_{ol})_{0^\circ C} \times (t^\circ C)}$$

- Pressure coefficient of a gas at constant volume per unit degree rise in temperature $\beta_P$ is given by:

$$\beta_P = \frac{P_{t^\circ C} - P_{0^\circ C}}{P_{0^\circ C} \times (t^\circ C)} = \frac{1}{273}$$

## Questions and Drills

**I) Complete (Fill in the spaces) :**

Which phrase (a-e) completes each of the next following statements (1-3)?

a) increases by a small value.

b) decreases by a small value.

c) remains constant.

d) doubles.

e) dereases to its half value.

1- If the pressure of a gas is doubled at constant temperature. So its volume...............

2- If a barometer is transferred to the top of a mountain above the sea level, the length of mercury in the barometer .............

3- If the absolute temperature of a gas is decreased to be half its original value at constant pressure, so its volume ...............

**II) Choose the correct answer:**

1- The increase of the temperature of a car's tire during motion leads to :

1) an increase in air pressure inside the tire.

2) an increase of air volume inside the tire.

3) a decrease of the contact area of the tire with the road.

Choose the correct letter (a-e)

a) (1, 2, 3) are correct.

b) (1, 2) only are correct.

c) (1, 3) only are correct.

d) 3 only is correct.

e) 1 only is correct

2- The temperature of a normal human body on the Kelvin scale is about:

a) 0°K b) 37°K c) 100°K

d) 373°K e) 310°K

3- The volume of a given mass of a gas is :

a) inversely proportional to its temperature at constant pressure.

b) inversely proportional to its pressure at constant temperature.

c) directly proportional to its pressure at constant temperature.

d) directly proportional to its temperature at variable pressure.

e) inversely proportional to its pressure at variable temperature.

4- The pressure of a gas at 10°C is doubled if it is heated at constant volume to :

a) 20°C b) 80°C c) 160°C

d) 293°C e) 410°C.

5- If we press a gas slowly to half of its original volume:

a) its temperature is doubled.

b) its temperature is decreased to half its value.

c) its pressure will be half of its original value.

d) the velocity of its molecules is doubled.

e) the pressure of the gas is doubled.

## III) Eassy questions

1- How can you show experimentally that the volume coefficient of expansion at constant pressure is the same for all gases?

2- Describe an experiment to find the pressure coefficient of a gas at constant volume and that it is the same for all gases.

3- How can you verify Boyle's law experimentally?

4- How can you show that pressure of a gas increases by raising temperature at constant volume?

5- How can you determine experimentally the absolute zero?

6- Explain the meaning of zero Kelvin and the absolute temperature scale.

7- Deduce the general gas law.

## V) Drills:

1- The temperature of one liter of gas is raised from 10°C to 293°C at constant pressure, find its volume. (2 liters)

2- A container containing air at 0°C is cooled to (-91°C). Its pressure becomes 40 cm Hg. Find the pressure of the gas at 0°C. (60 cm Hg.)

3- The volume of a quantity of oxygen at 91°C under 84 cm Hg is 760 $cm^3$ (S.T.P). Find its volume at 0°C under a pressure of 76 cm.Hg. (630 $cm^3$)

4- A flask containing air is heated from 15°C to 87°C. Find the ratio between the volume of air that goes out from it to its original volume. (25%)

5- A tire contains air under pressure 1.5 Atm at temperature (-3°C). Find the pressure of air inside the tire if the temperature is raised to 51°C, assuming that the volume is constant. (1.8 Atm)

6- An air bubble has a volume of 28$cm^3$ at a depth of 10.13 m beneath the water surface. Find its volume before reaching the surface of the water, assuming that the temperature at a depth of 10.13 m, is 7°C and that at the surface is 27°C.

(Let $g = 10ms^{-2}$ , $P_a = 1.013 \times 10^5$ N/$m^2$, $\rho = 1000$ kg/$m^3$) (60 $cm^3$)

المواصفات الفنية

| مقاس الكتاب | ٨٢x٥٧ سم |
|---|---|
| ألوان الكتاب | ٤ لون |
| ألوان الغلاف | ٤ لون |
| نوعية ورق الغلاف و وزنه | ٢٠٠ جم كوشيه |
| نوعية ورق المتن ووزنه | ٨٠ جم أبيض |
| عدد الصفحات بالغلاف | ١٢٤ صفحة |
| رقـــم الكتـــاب | ٩٤٦/١٠/١٥/٣٣/٢/٢٧ |

